

ARTS2062

5i glfU`]Ub'7]bYa U

HYfa "Hk c "#" &\$&%

7ci fgY'8YW]`g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

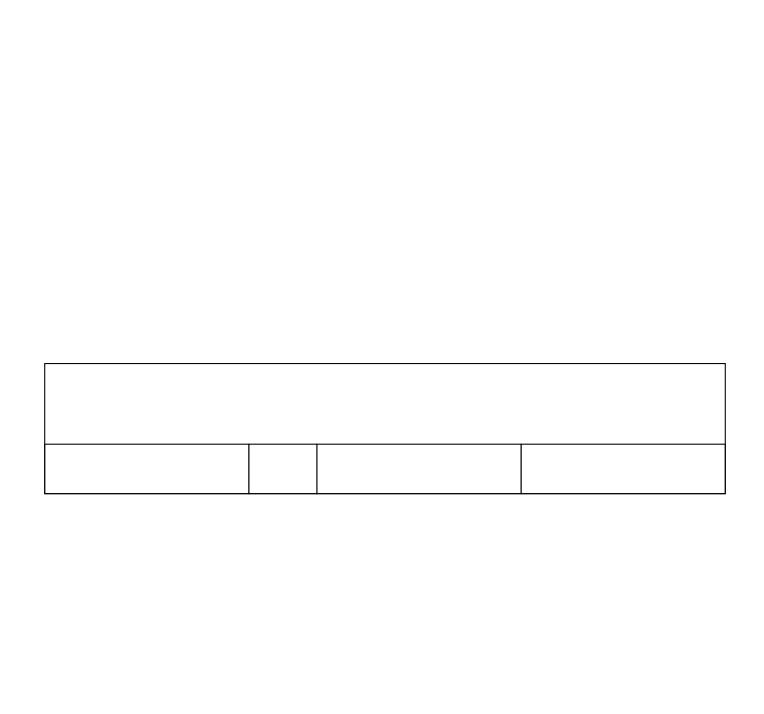
H\]g`Wci fgY`lfUWYg`h\Y`Ya Yf[YbWY`UbX`g][b]Z]WUbWY`cZ5i glfU`]Ub`W]bYa U`W``hi fY'''Mci `k]```ghi Xmlh\Y XYj Y`cda YbhcZh\Y`5i glfU`]Ub`Z]`a `]bXi glfmž`]bWi X]b[`UbU`mg]g`cZh\Y`YWcbca]Wz`gcW]U``UbX`dc`]h]WU`
ZJWcfg`UbX`h\Y`a ml\g`k \]W\`\Uj Y`g\UdYX`h\Y`]bXi glfm''Mci `k]```U`gc`ghi Xmlh\Y`fc`Y`cZh\Y`5i glfU`]Ub
gWfYYb`]b`g\Ud]b[`h\Y`YI dYf]YbWY`cZa cXYfb`5i glfU`]U`UbX`]hg`d`UWY`]b`h\Y`k cf`X"`H\Y`Wci fgY`U`gc
YI d`cfYg`U`fUb[Y`cZa U]bglfYUa `UbX`bcb!a U]bglfYUa `Zcfa g`cZUi X]c!j]gi U``dfcXi Wf]cb`]b`5i glfU`]U"H\Y
hcd]WgzZ]`a g`UbX`Wf]h]WU``HYI hg`\Uj Y`U```VYYb`gY`YWYX`hc`YbWci fU[Y`h\Y`ghi XmcZU`k]XY`fUb[Y`cZ
5i glfU`]U'g`bUh]cbU``W]bYa U"`

7 ci fgY @YUfb]b[Ci hWca Yg

- %" → XYbhjZmigca Y`cZh\Y`\]ghcf]\W\`žY\Wcbca]\Wzgc\\]U``UbX`\W`hi fU``\WcbhYl hg`cZ5i ghfU`]Ub`\\]bYa U &" 9j U`i UhY`h\Y`d`U\W`cZ5i ghfU`]Ub`\\]bYa U`]b`fY`Uh]cb`hc`< c``mk ccXžfY[]cbU`žUbX`ch\Yf`bUh]cbU`\\WbYa Uq
- '" =XYbhJZmiUbX'WfJhJWU``miUggYgg'h\Y'X]j Yfg]lmicZ5i glfU`]u'g'bUhJcbU`'WJbYa U'UbX'dfYgYbhmci f'ZJbX]b[g]b'Wc\YfYbhzk Y```glfi Whi fYXz'cfU`'UbX'k f]lhYb'Zcfa"

HYUWX]b['GhfUhY[]Yq'

H\Y`Wci fgY`k]```VY`HJi [\hi g]b[`'U`Wca V]bUh]cb`cZ`YWi fYgž'gWfYYb]b[gž'WJgg'X]gW gg]cbž'[fci d`fYgYUfW\UbX`dfYgYbHJh]cbž']bX]i JXi U``fYgYUfW\UbX`k f]h]b["'`H\Y`hYUW\]b[`'UddfcUW\'U``ck g`gh XYbhg`hc`Yl dUbX



XYYa YX``UhYžUbX`U`dYbU`hmUdd`]YX"'±i]g`h\Y`fYgdcbg]V]`]hmcZYUW\`]bX]j]Xi U``gh XYbhhc Ybgi fY h\UhUbmk cf_`]g`qi WWYqqZ```mqi Va]hhYX`VmXYUX`]bYžUbX`]b`]hg`WcffYWhZcfa Uh# Yfq]cb"

: cf'a cfY`]bZcfa Uf]cb'cb``UfY'dYbU`f]Yg`]b`G5A ž'UbX'cf\Yf'dc`]W]Yg'UbX`[i]XY`]bYgqYY.`\fida.#\da "Uffa"i bak "YXi "Ui #afi XYbfa#Yaci fWYa#dc`]W]Ya![i]XY`]bYa#

F Yei Yghg'Zcf'Yl hYbg]cb'bYYX'hc'[c'h\fci[\'GdYW]U` 7 cbg]XYfUhcb.'\hda.#gh XYbh'i bgk "YXi "Ui #gdYW]U!Wcbg]XYfUhcb'

⊋mci 'bYYX'hc'Udd`m'Zcf'GdYWJU``7 cbg]XYfUh]cb'd`YUgY'Udd`m'Uh`YUgh'U'ZYk 'XUmg'VYZcfY'h\Y'UggYgga Ybh Xi Y'XUhY"

Gi Va]gg]cb bch/g. Cbmcb/ gi Va]gg]cb U`ck YXždYUgY i d`cUX h\Y WcffYWhZ]Y

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

5ggYgga Ybh& A U'cf'9ggUmcf:]`a : Ygh]j U`7i fUh]cb

GhUfhiXUhY. Bch5dd`]WUV`Y

@Yb[h.:&\$\$\$`k cfXgž'bch']bWi X]b['ZcchbchYg'UbX'V]V`]c[fUd\miVi h']bWi g]j Y`cZ]b!hYl h'fYZYfYbWYg`fl%\$i cj Yf'UbX'i bXYf'fi `Y`Udd`]Ygž'% \$\$'!`&&\$\$`k cfXgŁ

8 YHJ]`g.

9 h\ Yf

9ggUm 'U'% \$\$!&\$\$\$!k cfX'YggUmcb'U'hcd]WZica 'U'`]ghidfcj]XYXžX]gWigg]b['\ck 'U'gdYW]Z]WZ]`a 'cf Zi`a a U_Yf'UXXfYggYg'h\Y'X]j YfgY'UbX''ci lk UfX!`cc_]b['bUh fY'cZ5i glfU']Ub'W]bYa U"'Gh XYbhg'k]```Z]fgh gi Va]hU') \$!%\$\$'k cfX'UVglfUWici h]b]b['h\Y]f'fUh]cbU'Y'Zcf'h\Y'Zi`a 'cf'Z]`a a U_Yf'h\Ym\Uj Y'gY`YWYX"'H\Y j U'i Y'cZh\Y'UVglfUWi]g'%\$i 'cZh\Y'hchU``a Uf_'Zcf'h\]g'UggYgga YbhihUg_"'Gi Va]lhYX'cb`]bY'j]U'@A G" : YYXVUW_UbX'U'bi a Yf]WU``[fUXY'dfcj]XYX'j]U'@A G" Cf

:]`a : Ygh]j U``7 i fUh]cb. ``U'%, \$\$!&\$\$\$!k cfX`dfc[fUa 'Zcf'U'k YY_YbX`Z]`a 'ZYgh]j U``cZ) '5 i ghfU`]Ub`Z]`a g`cb`U hcd]WZfca `U``]ghdfcj]XYX"'Ghi XYbhg`k]```Z]fghigi Va]hU') \$!%\$\$'k cfX``UVghfUWhci h`]b]b['h\Y]f`fUh]cbU'Y`Zcf'h\Y Z`a g'h\Ym\Uj Y`gY`YWhYX"'H\Y`j U`i Y`cZh\Y`UVghfUWh]g'%\$i 'cZh\Y`hchU``a Uf_`Zcf'h\]g'UggYgga YbhhUg_" Gi Va]hhYX`cb`]bY'j]U`@A G": YYXVUW_UbX`U'bi a Yf]WU`[fUXY`dfcj]XYX'j]U`@A G"

5XX]h]cbU`XYhU]`g.

D`YUgY`fYa Ya VYf`h\Uh'mci `a i gha U_Y`U`gYf]ci g`UhhYa dh`Uh`U```h\Y`UggYgga Ybhg`hUg_g`Zcf`h\]g`Wci fgY hc`VY`UV`Y`hc`dUgg`h\Y`Wci fgY"

5```a Uf_g'UfY'dfcj]g]cbU``i bhj``th\Ym\Uj Y'VYYb'Zcfa U``m'WcbZ]fa YX'Vmth\Y': UW'`tm'5WUXYa]WE i U`]tm 7 ca a]thYY'Uhth\Y'WcbWi g]cb'cZYUW\'gYa YghYf''`A Uf_g'a Um'VY'gi V'YWhc'W\Ub[Y'Zcf'j Uf]ci g'fYUgcbg]bWi X]b['th\Y'Udd`]WUh]cb'cZ`UhY'dYbU`th]Ygža cXYfUh]cb'UbX'th\Y'Udd`]WUh]cb'cZUWUXYa]Wa]gWcbXi Wh dYbU`th]Yg"

5ggYgga Ybhgʻa i ghVYʻgi WWggZi ``mgi Va]hhYX`Vmh\YʻgdYWJZJYX`Xi Y`XUhYʻUbX`hJa Yžcf`]hk]``VY XYYa YX``UhYžUbX`UʻdYbU`hmUdd`]YX'''±n`]gʻh\YʻfYgdcbg]V]`]hmcZYUWX`]bX]j]Xi U``gh XYbhhc Ybgi fY

h\UhUbmk cf_]g gi WWggZ ``mgi Va]hhYX VmXYUX`]bYžUbX]b']hg WcffYWhZcfa Uh# Yfg]cb"

 $: cf'a \ cfY']bZcfa \ Uf]cb'cb'`UfY'dYbU'f]Yg']b'G5A \ \check{z}'UbX'cf\,Yf'dc`]W]Yg'UbX'[i]XY`]bYg gYY. `\ \underline{fdg.} \ \underline{fW}g\#Ua \ \underline{Uffg''i} \ bgk \ \underline{YXi''Ui} \ \underline{\#ghi} \ XYbfg\#Ygci \ \underline{fW}'g\#Uc`]W]Yg![i]XY`]bYg\#$

F Yei Yghg'Zcf'Yl hYbg]cb'bYYX'hc'[c'h\fci[\'GdYW]U`7cbg]XYfUh]cb.'\hdg.#gh XYbh'i bgk "YXi "Ui #gdYW]U'!Wcbg]XYfUh]cb'

⊋mci 'bYYX'hc'Udd`m'Zcf'GdYWJU`'7 cbg]XYfUh]cb'd`YUgY'Udd`m'Uh`YUgh'U'ZYk 'XUmg'VYZcfY'h\Y'UggYgga Ybh Xi Y'XUhY"

Gi Va]gg]cb bch/g. Cb mcb/ gi Va]gg]cb U`ck YXzd YUgY i d`cUX h\Y WcffYWhZ]Y

 $\label{eq:higher} \textbf{Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh']g'gi Va]hhYX'h\fci [\''Hi fb]hjb'UbX'ghi XYbhg'WUb'gYY'Hi fb]hjb'g]a]`Uf]hm fYdcfhg''$

5 HMV BUXUUD VVV F YEE YEE YEVA Y b hg

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'h]a YhUV'Y

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

 $H Y f Y "] g "bc "h Y I h V cc _ Vi h Y U W "k Y Y _ "h Y f Y "U f Y "f Y e i] f Y X "f Y U X] b [g "U b X " _] a g "hc "j] Y k " "G h X Y b h g "U f Y "f Y e i] f Y X h Y "f Y U X] b [g "U b X "k U h W "h Y " _] a g "gc "h U h h Y m W b "W b h f] V i h Y " _] "m] b "h h c f] U "g "$

 $GYY'A ccX`Y`UbX`@Y[Ubltc`Zcf'lh\Y`fYei]fYX`UbX`fYWcaaYbXYX`fYUX]b[g`UbX`gWfYYb]b[g`U`gc`Zcfgi[[YghYX`fYgYUfW\`fYgcifWfg"`5`gc`gYY`A ccX`Y`UbX`@Y[Ubltc`Zcf'fYWcaaYbXYX`Z]`ag`UbX`lhY`Yj]g]cb$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK