

IEST5003

5XXfYgg]b['9bj]fcba YbHJ``=ggi Yg

HYfa 'C bY''##'&\$%

7ci fgY⁻Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

BUa Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
DUi ``6fck b	g	WcbhJWh"A YYh]b[g	A cfj Yb 6fck b 6i]`X]b[A 6' *)	\$(%(',))+\$

GWXcc``7cbHJWhi±bZcfa Uh]cb

GW.cc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@c\WJh]cb. G\Wcc``C\Z]\\YžA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&\z'&),

CdYb]b['< ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY. 'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-' ,) ', +\$)

9a U]`. '\U`4 i bgk "YXi "Ui_

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y'7ci fgY

H\]g`Wci fgY`]`i glfUhYg`h\Y`bUhi fY`cZz`UbX`bYYX`Zcfz`a i `h]! `cf`lfUbg!X]gW]d`]bUfmIUddfcUWXYg`Zcf`UXXfYgg]b[
Ybj]fcba YbhU``dfcV`Ya g`cZbUh]cbU``UbX#cf`[`cvU``]a dcflUbW!"; fci d`k cf_gYa]bUf`dfYgYbhUh]cbg`k]``
XfUk `cb`W ffYbhcf`fYW'bh_YmYbj]fcba YbhU``]ggi Yg`UbX`k]```VY`gi ddcfhYX`VmVf]YZ`YWfi fYg"'Gca Y`cZh\Y
hcd]Wg`mci `k]```Wcj Yf`UfY. Ybj]fcba YbhU``[cj YfbUbW\zi]bhYfbUh]cbU``Ybj]fcba YbhU``fY[]a Yg#\cbj Ybh]cbg/
ZfUa]b['UbX`XYZ]b]b['Ybj]fcba YbhU``]ggi Yg/di V`]WdUfh]\w]dUh]cb`UbX`ghU_Y\c`XYf`Yb[U[Ya Ybh]b`XYW]g]cb
a U_]b[/]bhY[fUhYX`UbX`Zcfk UfX``cc_]b[`UbU`mg]g/d`Ubb]b[`Zcf`gi ghU]bUV`Y`XYj Y`cda Ybh'g\w]YbW\zif]g__UbX
_bck `YX[Y/Wca d`Yl `gmghYa g`UbX`UXUdh]j Y`a UbU[Ya Ybh'gi ghU]bUV`Y`bUhi fU``fYgci f\w`a UbU[Ya Ybh'Ybj]fcba YbhU``]a dU\wg`cZdfcXi Wh]cb`UbX`Wcbgi a dh]cb"

7ci fgY @YUfb]b[Ci hWca Yg

%" 8Ya cbghfUhY`Ub`UXj UbWYX`i bXYfghUbX]b[`cZh\Y`a i `h]!X]gW]d`]bUfmVcXmcZ_bck `YX[Y`bYYXYX`Zcf Ybj]fcba YbhU``a UbU[Ya Ybh'UbX`Udd`mX]gW]d`]bUfmUbX`hfUbg!X]gW]d`]bUfmdf]bW]d`Yg`UbX`dfUWf]WYg hc`bYk `cf`Wca d`YI `W\U``Yb[Yg"

&" $8 \text{ Yj Y'cd'} \text{UbU'm'j} \text{WU''g}_j \text{"gž'} \text{UXj UbWYX'} \text{Wf} \text{Jh} \text{JWU''} \text{HUb}$

‰

ghi XYbhgʻUfYʻYl dYWYXžcbʻk YY_`mlVUg]gžhcʻk f]hYʻUʻVʻc[ʻYbhfmfYZYW]b[ʻcbʻh\Y]fʻ`YUfb]b[ʻcZh\Yʻk YY_`mhcd]Wʻ8]ghUbWʻghi XYbhgʻk]```UʻgcʻhU_YʻfYgdcbg]V]`]mnZcf``YUX]b[ʻžYbWci fU[jb[ʻUbX'a cXYfUh]b[ʻcb`]bYX]gWʻgg]cbʻcj YfʻcbYʻZ ```k YY_žcbʻh\Yʻk YY_`mi b]hhcd]WUbX#cf`WUgYʻghi XmlʻH\Ymk]```dfcXi WYʻUbʻcd]b]cb YggUmUgʻh\YʻZ]bU``Ugg][ba YbhZcfʻh\Y`Wci fgY"

5ggYgga Ybh

: i fth Yf [i]XUbWY 'Uj U] `UV Y 'j]U 'th Y 'Wci fgY 'A ccX Y 'g] th "

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
K YY_`m'6`c[·	' \$ı	Xi Y YUWX 'k YY_	%ž&Ž ž(ž)
:]Y`X`5gg][ba Ybh	&\$ı	8 i Y a]X hYfa žYl UWhXUhY hc VY UXj]gYX	%ž&Ž ž(ž)
; fci d'7 UgY Gh Xm	&\$ı	Hck UfXg'h\Y'YbX'cZhYfa " 8 UhY'H65"	%ž&Ž Ž(Ž)
Cd]b]cb 9ggUm	' \$ı	8i Y'Uhh\Y'YbX'cZhYfa "'8UhY H65"	%ž&Ž Ž(Ž)

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% K YY_`m6`c[

GhUfh'XUhY.

 $@Yb[h \ . \ '; \ YbYfU``m`Ufci bX'' $$'k cfXg'Zcf'YUWX'V`c[" \ . \ ']$

8 YHJ]`g. 'K YY_`m`g\cfhk f]hhYb`gi a a Uf]Yg`Zcf`X]ghUbWY`UbX`cb!WUa di g`ghi XYbhgž'k]h\`g\cfhicb`]bY ZYYXVUW_[] jj Ybž'Zcf`%\$`k YY_g`r' \$\$`k cfXg`dYf`k YY_

5XX]h]cbU`XYhJ]`g.

Hcd]\Mg^Zcf^mcif^V^c[g^k]^``VY^YgHUV^]g\YX^YUW\kYY_žUbX`k]^``fY^UHY^bc^mcif^]bXYdYbXYbhfYgYUfW\"

Gi Va]gg]cb bchYg.Gi Va]hk YY_mj]U mci f'A ccX'Y g]hY

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh'&:]Y`X'5gg][ba Ybh

Ghufhixuhy. + h\y Zfghh\fyy k yy_g cZh\y Wci fgy

@Yb[h\.'%\\$\$\$\kcfXg

8YHU]`g. `Gh XYbhg`k]```VY`fYei]fYX`hc`gi Va]hU`%\$\$\$\$k cfX`fYdcfhVUgYX`cb`U`Z]Y`X`Ugg][bYa Ybh

 $\begin{tabular}{ll} \textbf{Gi Va]gg]cb`bch'g. Mci f'dfcdcgU``Zcf'mci f'Z\Y`X'k cf_ik]```VY'cbY'cZmci f'V`c[g''`H\Y`Z\bU``fYdcfhik]```VY gi Va]\text{h'}X'j]\text{U'mci f'A ccXY'g]}h'Y \end{tabular}$

 $\label{eq:higher_problem} \mbox{Hi fb]h]b "gYhh]b[. "H\]g"]g"bchU"Hi fb]h]b"Ugg][\mbox{ba Ybh}$

5Hxh\$

Mk V\$ Uaaah Ud duvfy5f mint i U[yah "U" "U" Uaaya %Us if Nk

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

±JZcfa Uh]cb'UVci hfYgci fWyg'k]``VY'j]U'h\Y'Wci fgY'A ccX`Y'g]hY"'FYUX]b[g'k]``VY'`]ghYX'UbX'a UXYUjU]`UV`Y'h\fci [\'h\Y'@9; 5BHC'gmghYa"

FYWca a YbXYX FYgci fW/g

 $\pm \sqrt{2} cfa \ Uhjcb 'UVci \ hfYgci \ fW'g'k] ```VY'j] U'h\Y'Wci \ fgY'A \ ccX`Y'g]hY'''F YUX]b[g'k] ```VY``]ghYX'UbX'a UXY Uj U]`UV`Y'h\fci [\'h\Y'@9; 5BHC 'gmghYa ''`\Mci 'k]```U`gc'VY'YI dYWYX'hc'WcbXi WhjbXYdYbXYbhfYgYUfWk h\Uh`YUXg'mci 'hc'ch\Yf'gdYW]U`]gYX'gci fW'g''$

7 ci fgY 9j U`i Uhcb UbX 8Yj Y`cda Ybh

A m91 dYf]YbWY'k]``VY'i gYX'Uh'h\Y'YbX'cZgYgg]cb'hc'Yj U`i UhY'h\Y'Wti fgY"

: cWg [fci dg k] ``dfcj [XY UXX] hcbu `ZYYXVUW_hck UfXg h Y a [XX Y cZh Y Wci fgY"

6 UgYX'cb'dUghghi XYbhYl dYf]YbW'UbX'fYgi `h]b['Wci fgY'XYj Y`cda Ybhž'h\Y'ZcW'g'k]``VY'cb'UddfcUW\Yg h\Un'ZcghYf']bXYdYbXYbhfYgYUfW\'UbX'k f]n]b['h\Un']g'cZU'gW\c`Uf'm'\][\'ghUbXUfX/UbX'ci hdi hg'h\Un'UfY'cZ dfUW]\W]`i gY']b'Z]bX]b['Ybj]fcba YbhU`'gc`i h]cbg"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWUb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU``mWcdm]b['Zfca `U`gci fW'k]h\ci hUW_bck `YX[Ya Ybh'

 $I BGK `[fci dg'd`U[]Uf]ga `]bhc'h\Y'Zc``ck]b[`WUhY[cf]Yg.$

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

5 WUXYa W + DZcfa Uhlcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb'fY`UhJb[hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'th 'UbX'GUZYlmidc']W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUljcb`]b`l\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]mrUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W']bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

=a U[Y'7fYX]h

6UfUb[Ufcc. `GmXbYmfg'i fVUb`fY!XYj Y`cda YbhWkU``Yb[Yg'i g'hc'hk]b_'UVci hUXUdhj Y'UbX'dUfhjWjdUhcfm Ybj]fcba YbhU``a UbU[Ya Ybh]b'hkY'ZUWY'cZdfYgg]b['Ybj]fcba YbhU`]ggi Yg'-'\ck 'hc'WcbhU]b'UbX'hfYUh WcbhUa]bUhYX']bXi ghf]U``g]hYgž\ck 'hc'Hu_Y'UWWci bhcZW 'h fU`\Yf]hU[Y']bWi X]b['±bX][Ybci g'\Yf]hU[Y'UbX]bXi ghf]U``\]ghcfmž\ck 'hc'Yb[U[Y'fdi V`]Wgfi]b'Ybj]fcba YbhU``XYWJg]cb!a U_]b["'D\chc[fUd\'VmDUi ``6fck b"

7F = 7CG

7 F = 7 C G 'Dfc|]XYf'7 cXY. '\$\$\$-,;