

ARTS3633

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Gi VYWh5fYU. '<SbS` WeWEfgV[We

H\Y'Wei fgY'\Ug'h\fYY'd\UgYgžk\]W\fY`UhY'hc'U'fYgYUfW\dfc YWhcb'h\Y'>UdUbYgY``Ub[i U[Y']b'cfXYf'hc XYj Y`cd'mci f'`]b[i]gh]W_bck `YX[Y'UbX'g_]``g']b`XYdh\"'\#o'h\Y'Z]fghd\UgYžh\Y'Wei fgY``YUXg'mci 'hc'Yl Ua]bY UbX'X]gW gg'fYUX]b[g'UbX'j]gi U``a UhYf]U`g'dfYgYbh]b['j Uf]ci g'UgdYWg'cZh\Y'W\UfUWYf]gh]Wg'cZh\Y'\UbX'Yl dUbX'mci f'_bck 'YX[Y'cZ[fUa a Ufžj cWUVi `UfmžYl dfYgg]cbgž] S`\\Z'UbX X]ZYfYbhgmiYg'UbX'fY[]ghYfg']b'>UdUbYgY'h\fci [\`]bhYfUW]jY'Y'l YfW]gYg']b'Vch\``YWh fYg'UbX'h rcf]U`g" H\YgY'dfUW]Wg'dfcj]XY'mci 'gca Y'\]bhg'UbX']XYUg'hc 'cdhZcf'mci f'fYgYUfW\hcd]W"\#o'h\Y'gYWcbX'd\UgYž mci 'dfYdUfY'U'dfc YWhVmYl d`cf]b['U'gdYW]Z]WUfYU'cZ>UdUbYgY'mci 'Z]bX']bhYfYgh]b['cf'i gYZ\`''5ZhYf dfUW]W]b['Wea a i b]Whfcb'gHUhY[]Yg'UbX'g_]``gž'U'gYf]Yg'cZ]bhYfj]Yk 'gYgg]cbg'Zcf'XUhU'Wc``YW]cb'[]j Y'mci cddcfhi b]h]Yg'hc]bhYfUWhk]h\'mci f'dYYfg'UbX'bUh]j Y'>UdUbYgY'gdYU_Yfg''\#o'h\Y'Z]bU`'d\UgYžmci 'k f]hY'U dUdYf'UbX'dfYgYbh]h]b'>UdUbYgY''Nci i gY'mci f'UbU`mi]WD'`UbX'Wf]hWD'`h\]b_]b['Ug'k Y``'Ug'dfYgYbhUh]cb UbX'UWIXYa]Wk f]h]b['g_]``g''H\]g'Wci fgY']g'hu [\hi]b'>UdUbYgY''

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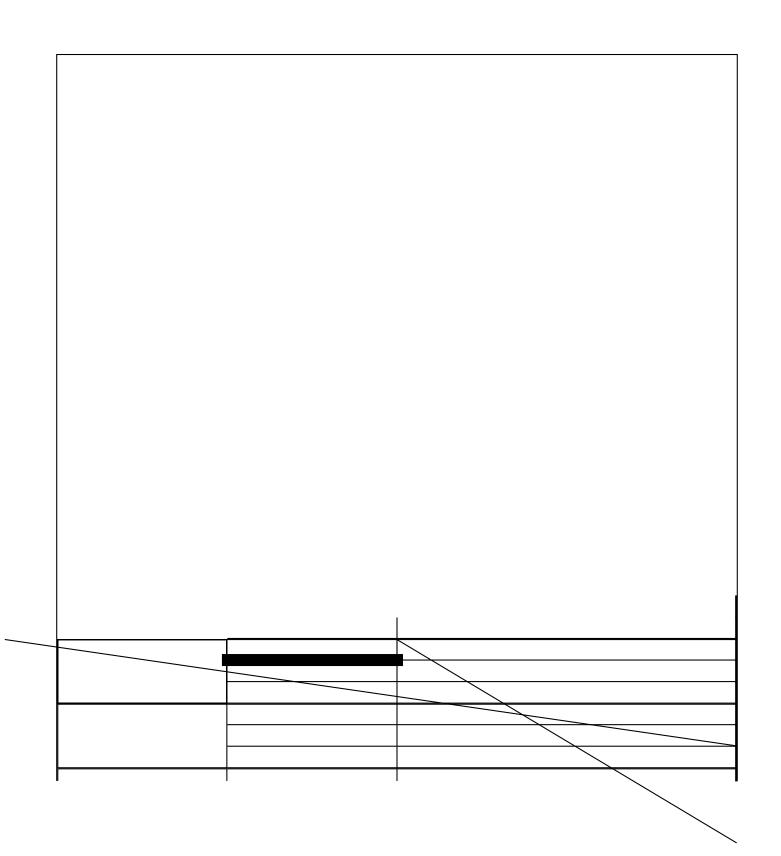
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FYWca a YbXYX FYgci fWYg

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H\Y`Wci fgYg`]g`dYf]cX]WJ``mfYj]Yk YX`UbX`ghi XYbhg'`ZYYXVUW_`]g`i gYX`hc`]a dfcj Y`]hg`WcbhYbhg`UbX UWijj]hjYg'': YYXVUW_`]g`[Uh\YfYX`Zfca `ghi XYbhg`i g]b[`a m9| dYf]YbWY''=n]g`YbWci fU[YX`h\Uhghi XYbhg Wca d`YhY`h\Y]f`gi fj Ymg`VmUWWgg]b[`h\Y`dYfgcbU`]gYX`k YV``]b_`j]U`h\Y`A ccX`Y`Wci fgY`g]hY''

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a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

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7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

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- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@G9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.#\gi \YY\\f\i \\XY\g''\\f\f\f\i \bgk\\\YX\i \"\J\\Y\\\f\\\]aY