

ARTS3458

 $7\] bYgY`7\UdghcbY.`FY!J]g]cb]b[`7\]bU`UbX`7\]bYgYGh X]Yg$

HYfa 'H\fYY''##'&\$&\$

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
K U\`; i Ub`@]a		5	A cfj Yb 6fck b &()	-',)'%,*%

GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . 'Ž * %'&'-' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY 8YW] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y'7ci fgY

H\]g'Wci fgY'Yb[U[Yg'k]h__Ym]XYUg'UbX']ggi Yg']b'7\]bYgY'Ghi X]Yg''=iU'gc'fY!j]g]cbg'UfYUg'h\Uh\Uj Y VYYb'Wcj YfYX']b'ch\Yf'Wci fgYg']b'h\Y'A U'cf''H\YgY']bWi XY'7\]bYgY'`Ub[i U[Y'ghi X]Ygž']hYfUhi fYžZ]`a ž a YX]Už'hfUbg`Uh]cbžUbh\fcdc`c[mUbX'[YbXYf'ghi X]Yg''Mci 'UfY'YbWci fU[YX'hc'fYZYWhi dcbžfYj]Yk 'UbX i gY'h\Y_bck 'YX[Y'UbX'`Ub[i U[Y'Wca dYhYbW'UWei]fYX'Zfca 'h\YgY'UfYUg'cZghi XmUbX'Wcbg]XYf'\ck 'mci WUb'i hj`]gY'h\Ya 'Zcf'mci f'UWUXYa]Wcf'WUfYYf'Z hi fYg'''Mci 'k]``[U]b'U'a cfY'h\YcfYh]WU`'UbX'\]ghcf]WU` dYfgdYWhj Y'cb'7\]bU'UbX'7\]bYgY'Ghi X]Yg'''H\Y''YUfb]b['k]``'VY'ZJWj]hUh'X'VmU'fUb[Y'cZUWhj]h]Yg]bWi X]b[']bXYdYbXYbhfYgYUfW\dfc YWgžcfU`'dfYgYbhUhjcbg'UbX'YggUmk f]h]b['''H\fci [\h\Y'Wci fgYžmci UfY'hc''YUfb'UbX'fYZYWhcb'h\Y'WcbhYl hgža Yh\cXg'UbX'\]ghcf]Yg']b'k\]W\'7\]bYgY'Ghi X]Yg'\Ug'VYYb UddfcUWXYX/hc'[U]b'Ub']bZcfa YX'i bXYfghUbX]b['cZh\Y'_Ym]ggi Yg'UbX'XYVUhYg'UVci hWcbhYa dcfUfmUbX\]ghcf]WU`'7\]bUJ/hc'fYj]Yk'UbX'i dXUhY'gWxc`Ufg\]d'cb'7\]bYgY'Ghi X]Yg'Zfca 'Ub']bZcfa YXžWf]h]WU`'UbX]bhYfX]gW[d']bUfmdYfgdYWhj Y''

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 5 Wei]fY _bck `YX[Y]bXYdYbXYbhmži hj`]g]b['Vch\ df]a UfmUbX gYWcbXUfma UhYf]U`g
- &" 8Ya cbghfUhY`WcbWYdhi U``UbX`dfUWj\WJ``g_]``g`Zcf`Wf]hJ\WJ``Ybei]fmiUbX`dfcV`Ya `gc`j]b[`cb`]ggi Yg`cZ XYVUhYq
- ' "D`Ub`UbX`Wcad`YhY`U`fYgYUfW\`dfc^YWh
- ("5fh]W'`UhY`gdYWJU`]gYX`_bck`YX[Y`cZ7\]bU`UbX`UgdYWfg`cZ7\]bYgY`Gh X]Yg
-) " 9b[U[Y]b fYUgcbYX UbX cdYb!a]bXYX X]gW gg]cb UbX XYVUlY

HYUWX]b[GhfUhY[]Yg

 $\label{lem:lem:hamiltonian} $$H\]g'WJdghcbY'WcifgY']g'h\Y''UghWcifgY'Zcf'gh'XYbhg']b'h\Y''7\]bYgY'Gh'X]Yg'UbX'5Xj UbW'X''7\]g'WJdghcbY'WcifgY']g'h\Y''UghWcifgY'Zcf'gh'XYbhg']b'h\Y''7\]$ Gh X|Yqa U'cfq" +i]q XYq|| bYX hc dfc| |XY Ub cddcfh b|lmZcfqh XYbhq hc fYZYWicb h\Y|fYl dYf|YbWrq UbX _bck `YX[Y`]b`7\]bYgY`Gh X]Ygžhc Wcbgc`]XUhY`k\Uhh\Ym\UjY``YUfbhdfYj]cig`mžUbX`hc`dfYdUfY`h\Y]f lfUbg]hicb ibhc h∖y k cf ZcfW cfdcahi fUXi Uhy ah Xm" ±iibhy [fUhya bck `YX[Y [UibyX h\fci [\idfy] ci a Wci fqYk cf_'UbX'cj YfqYUq'Yl dYf]YbW'UbX'Vi]`Xg'cb'h\Uh'WcbW'dh U`'Zci bXUhcb'h\fci [\'U'a cfY h\YcfYh]\WU``UbX`\]qhcf]\WU``dYfgdY\Mjj Yž]\bhY[fUhjj Y`UbU`mg]qž'dfU\Mj\WU``Udd`]\WUhjcb`UbX`\Mj]hJ\WU``h\]b_]b["`H\Y Wci fqY']g'Wca dcgYX'cZU'gYf]Yg'cZh\Ya YX'`YWi fYgž']b!WUgg'X]gW gg]cbgžk cf_g\cdg'UbX'dfYgYbhUh]cbg"'+h]g'VUgYX'cb'h\Y'df]bWjd'Y'cZgh'XYbhqfigY'Z[i]XYX''YUfb]b["'H\Y'fc'Y'cZh\Y'WcifgY'WccfX]bUhcf'UbX''YWhifYf]g'hc'WfYUhY'Ub']bhY``YWhU`mgh]ai`Uh]b['UbX'WccdYfUh]jY'`YUfb]b['Ybj]fcbaYbhZcf'ghXYbhg'hc'Yb[U[Y']b X]U`c[i Yg`k]h\`YUVX`ch\Yf`]b`UbX`ci hcZWUggfcca ž'VYg]XYg`dfcj]X]b[`Ub`]bhY``YWhU``\]ghcf]WU``dYfgdYWhj Y UbX'gdYWJU`]nYX'X]gWJd`]bUfmZfUa Yk cf_"'Gh XYbhg'UfY'YbWci fU[YX'hc`fYUX'YI dUbg]j Y`mUbX'ZcghYf'U dfcXi Whi Y fY Uhicbq\id k jh\ h\ Y VfcUXYf UWUXYa jWWca a i b jhm' @YWi fYq UbX hi hcfjU'g Uja hc VfUjbqhcfa UbX'[]i Y'gh XYbhg'cddcfh b]h]Yg'hc'fll£'i bXYfghUbX'UbX'UWei]fY'U'gdYW]U']gh_bck 'YX[Y'cZ7\]bUž'7\]bYgY W`h fYž\]qhcfmUbX``Ub[i U[Y/f&LfYi]q]hYUf]Yf``YUfb]b[YI dYf]YbW'qž'UbX`]XYbhJZmX]ZYfYbhqHYUa q cZ thci [\haxyj Y`cdYX`]b`dUff\W``Uf`UfYUg`]b`fYWYbhmYUfg/fl EYI d`cfY`X]gW\d`]bUfmUbX`]bHYfX]gW\d`]bUfm UddfcUWXYg'UbX'a Yf\cXc`c[]Yg']b'fYgYUfWX]b['dUff\W'\Uf'hcd]Wg']b'7\]bYgY'Gh X]Yg"

5ggYgga Ybh

8 YHJ]`gʻcbʻ5ggYgga YbhF Yei]fYa Ybhg.

Dfc YWn8 YgWf]dh]cb#9ggUmDfcdcgU`.

 $\label{thm:linear} $$ Mci 'bYYX'hc'gi Va]hU'bYUhmk f]hhYb'dfcdcgU'`XYgWf]V]b['h\Y'fYgYUfW\'dfc'YWhmci '\Uj Y'bY[ch]UhYX'k]h\ mci f``YWh fYfz'UbX'\ck 'mci 'd'Ub'hc'i bXYfhU_Y']h''=hg\ci `X']bWi XY'h\Y'Zc``ck]b['UfYUg.$

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5ggYgga Ybh'8YhJ]`g
5ggYgga Ybh% A ccX`Y Dcglg/ 8]gW gg]cb @YUXYf
GhUfhXUhY. Bch5dd`]WUV`Y
@Yb[h\."$$!)$$ k cfXg
8 YW] g.
K YY_`m'A ccX`Y`Dcghg'/ 'Uh``YUgh'cbY`k YY_'hc``YUX`X]gW gg]cb"
\pm 1.000
Gi Va ]gg]cb bch/g.Gi Va ]hcb A ccXY VmA cbXUmbccb df]cf hc h\Y k YY_cZX]gW gg]cb
Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh
5ggYgga Ybh'& Dfc YWh'8YgWf]dh]cb #9ggUmDfcdcgU`
GhUfhiXUhY. K YY_"
8 YHJ]`g.
6]V`]c[fUd\mfYgYUfW\%\/dfcdcgU``cZZ]bU``YggUm'
#>! WUgg 'ZYYXVUW_"
Gi Va ]gg]cb bch/g.GcZhWcdmhc VY gi Va ]hh/X Y YWfcb]WU``m
Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh
5ggYgga Ybh' . DfYgYbhUh]cb
GhUfh'XUhY.
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%) !a ]bi h/ df/gYbhJh]cbž) !a ]bi h/ X]gW gg]cb YUW."
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Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh
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5ggYgga Ybh (. F YgYUfW 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.")\$\$*kcfXg

8YHJ]`g.

') \$\$!k cfX YggUm

: UWY!hc!ZUWY a YYh]b['k]h\]bghfi Whcf'cb`mi dcb'fYei YghcZghi XYbh'

 $\label{thm:condition} \mbox{Hi fb]h]b"gYh]b" Low Ybh"]g"gi Va]hhYX"h\fci [\h]b" Low X"ghi XYbhg"Xc"bchigYY" Hi fb]h]b g]a]"Uf]hmfYdcfhg" \mbox{Hi fb]h]b" Low Ybh"]g"gi Va]hhYX"h\fci [\h]b" Low X"ghi XYbhg"Xc"bchigYY" Hi fb]h]b Low Ybh"]g"gi Va]hhYX"h\fci [\h]b" Low Ybh"]g"g [\h]b" Low Ybh"]g"g [\h]b" Low Ybh"]g"g [\h]b" Low Ybh"]g"g [\h]b" Low$

5 http://bxubww.F.Yei]fYa Ybhg

5 hh/bXUbW'cZGYa]bUfg']g'aUbXUhcfm]b'h\]g'WcifgYžk\]W\k]''VY'hU_Yb']b'dYfgcb'UhYjYfmaYYh]b["IbYIW'gYX'UVgYbW''Zfca'acfY'h\Ub'%lcZGYa]bUfg'k]``'fYgi`h']b'h\Y'UkUfX'U'ZU]''[fUXY"'+b'GYa]bUfgmci'k]``'fYgi`h']b'h\Y'mYb[U[Y'k]h\'WcfY'WcifgY'WcbhYbhz'YbUV']b['mci'hc'UhtU]b.

K YY_'+.'&*'CWcVYf'!''\$ CWcVYf		@Ub[iU[Y'UbX'¥Ybh]m Dfc^YWh8YgWf]dh]cb'#9ggUmDfcdcgU`'Ž 6]V`]c[fUd\mXiY'f1%)ıŁ
K YY_', .`&'Bcj Ya VYf'!'* Bcj Ya VYf	GYa]bUf	7]bYaUh]WUbX'h\Y'DYfZcfa]b['5fhg
K YY_''- 'Bcj Ya VYf'! % 'Bcj Ya VYf	GYa]bUf	Dfc^YWhdfYgYbHJh]cbg^UbX^X]gWigg]cb^fk)ı Ł
K YY_'%\$.'%*'Bcj Ya VYf'! &\$'Bcj Ya VYf	GYa]bUf	K f]h]b[k cf_g\cd

FYgci fWyg

DfYgWf]VYX`F Ygci fWYg`

FYei]fyX^UbX^fyWca a YbXyX^fyUX]b[g^cb^A ccX^Y

FYWca a YbXYX FYgci fWYg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhjb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\'Hi fb]h]bz'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'j UbhXcW a Ybhg\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k]'`

JbWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\wUh]cb"'\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k]'`

Ui hca Uh]\wU'`mfYW']j Y'U'h]\w_Yhbi a VYf\mathred{Image: hY'Yd\cbY'mci 'k]\tag{Y}\wu''mug_'\mathred{Image: Wa'\mathred{Image: Wa'\mathred{Image: hY'\mathred{Image: hY'\mathred{Image:

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUhjj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY