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- &" Yj U'i UHY'H\YgY'XmbUa]Wg'UbX'dfcWYggYg']b fY'UH]cb'hc'U'gdYW]ZWWci bhfmcf'k cf'X'fY[]cb"
- ' "]bHY [fUHY 'UbU`mt]\WJ``g_]``gžk f]h]b[`\WJdUV]`]h]YgždfcV`Ya `gc`j]b[`UbX`dfYgYbHJh]cb`g_]``g`]b`h\Y WcbHYI hcZ]bHYfbUh]cbU``ghi X]Yg"
- ("a UHWX 'dfcZYgg]cbU``Wca dYHYbW]Yg`UbX`WUfYYf`fY`Yj Ubh_bck `YX[Y`hc`dfUWf]WY`]b`]bhYfbUh]cbU` cf[Ub]qUh|cbg`UbX`Vi q]bYgqYq"

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H\]g`XY[fYY`WUdghcbY`Wci fgY`\Ug`VYYb``XYj Y`cdYX`hc`hU_Y`]bhc`UW\ci bhh\Y`CGD`YI dYf]Yb\Wg`cZghi XYbhg fcf`h\Y`Yei]j U`YbhYI dYf]Yb\Wf'=BGH' - \$\$`]g`hUi [\hcj Yf`\%\$`k YY_g`\cbg]gh]b[`cZU'' `\ci f`\ci fYci fgY`f&\f` Y\Wfi fY`UbX`\fi hcf]U``cb`_Ym]ggi Yg`UbX`\WU``Yb[Yg`]b`=bhYfbUh]cbU``Ghi X]Yg`"'H\YgY`UfY`ZfYY!Zck]b[X]g\W gg]cbg`\WbhYfYX`cb`dfcj c\Wh]j Y`hi hcf]U``ei Ygh]cbg`YU\W`k YY_ž'k \YfY`ghi XYbhg XYVUhY`h\Y`]ggi Yg UbX`Z]bX`gc`i h]cbg`hc`h\Y`dfcV`Ya g`UbX`\WU``Yb[Yg`cZci f`h]a YgžXfUk]b[`cb`h\Y]f`dfYj]ci g`' `mYUfg`cZghi Xm]b\Wi X]b[`\%mYUf`gdYbhUVfcUX`]b`hfUj Y``UbX`ghi XmXi f]b[`CGD"

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