

# **ARTS5505**

DYfgcbU`]gYX'9b[`]g\'@Ub[i U[Y'9b\UbWa Ybh

Gi a a Yf"#"&\$&%

## 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## GHJZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
5`]gU'H]Ub	30 3	D`YUgY`WcbHJWfij ]U Ya U]`		

### GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b[ '<ci fg. 'A cbXUm'! : f]XUmž - Ua '! ') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-' , ) ', +\$)

9a U]`. '<u>\U`4 i bgk "YXi "Ui</u>

## 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

7 ci fgY @YUfb]b[ Ci hWca Yg

## 5ggYgga Ybh

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
DYfgcbU`dfcYWiXYg][bfDD8Ł	' \$ı	%\$#\$%#&\$&%`%%\$\$`DA	'ž(ž)
Dfc^YWh`c[Vcc_'UbX'cifbU`'	&\$I	&-#\$%#&\$&%`%%\$\$`DA	%ž <sup>*</sup> &
Cfu``DfYgYblUljcb'!'%\$ a ]bi lYg'	&\$ı	+b`WUgg`cZ`Ughk YY_	(ž)
FYZYW¶jY'YggUm	' \$1	\$%#\$&#&\$&%`\$\$\$`5A</td><td>%ž &</td></tr></tbody></table>	

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% DYfgcbU dfc YWhXYg][ b fDD8Ł

@Yb[ h\.'%) \$\$ k cfXg

8YW]`q.

Ghi XYbhg XYg][ b h\Y]f ck b dYfgcbU`dfc YWhrc 'UXXfYgg'h\Y]f ck b '9b[`]g\``Ub[i U[Y'bYYXg"'H\YmUfY fYei]fYX'hc 'gdYV]fmh\Y]f 'dYfgcbU`dfc YWh[ cU'žVUW\_[ fci bXža Yh\cXžfYgci fWgždfc[ fYgg'a YUgi fYa Ybhžha Y`]bYžgW\YXi `Y'UbX'Ubh]V]dUhYX'ci hWca Yg"'H\YmbYYX'hc 'XYj Y`cd'h\Y]f 'DD8 'VUgYX'cb'fY`Yj Ubh `]hYfUh fY'cZh\Y']Y`X'WcbWfbYX"'H\Y'k cfX``]a ]h]g'%) \$\$"

Ghi XYbhgʻfYWY]j Y`k f]hhYbʻZYYXVUW\_`VUgYX`cb`fi Vf]Wg''`

 $\label{thm:continuous} \begin{tabular}{ll} His plant of the lighthfully a property of the ligh$ 

5ggYgga Ybh& Dfc YWh`c[Vcc\_'UbX' ci fbU`

8 YHJ] g.

 $Gh XYbhg fYWcfX h Y]f YUfb]b[ YYd fYZYWh] Y chifbU chiU fY[i YufYUg]g Ug d YbbYX ]b h Y]f DD8 "HYMUfY fYei]fYX h gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbX'' $'\ci fg'cf'a cfY Zcf'h Y]f dYfgcbU dfc YWh gdYbY fY gdYbY gdYbY$ 

Gh XYbhg fYWr] Y'k f]hhyb ZYYXVUW\_VUqYX cb fi Vf]Wg"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh' . CfU DfYgYbHJh]cb! % a ]bi hYg

GHUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %\$ a ]bi hYg f) !+ a ]bi hYg cZdfYgYbhUh]cb Ž ' !( a ]bi hYg cZE/5Ł

8 YW] g.

Ghi XYbhgʻdfYgYbhih\Y]f``YUfb]b[ `\][\`ngʻZfca 'h\Y]fʻdYfgcbU``dfcYWhZcfʻUVci h'%\$`a ]bi hYgʻhc`]bgd]fY dYYfgʻhc`Wcbh]bi Y`hc`Yb\UbWi`h\Y]f``Ub[i U[Y`g\_]``gʻYj Yb`UZhYfʻWca d`Yh]b[ 'h\Y`Wci fgY"'H\YmUfY`fYei ]fYX`hc gi Va ]hUb`UVghfUWhfUVci h'%) \$`k cfXgŁ`UVci hk\Uhh\YmUfY`[c]b[ 'hc`dfYgYbh'U'Wci d`Y`cZk YY\_g`VYZcfY`h\Y dfYgYbh'Uh]cb`XUm''Ghi XYbhgʻfYWY]j Y`k f]hhYbʻZYYXVUW\_Zfca 'hi hcf`cb`UVghfUWhg`fW\$i Ł`UbX`Zfca 'dYYfg`cb dfYgYbh'Uh]cb`f\%i Ł`VUgYX`cb`fi Vf]\\g''

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$ 

5ggYgga Ybh ( FYZYW) j Y YggUm

@Yb[h\. &\$\$\$ k cfXg

8YHJ]`g.

Gh XYbhgʻUbU`mh]WU``mUbX`Wf]h]WU``mfYZYWhcb'h\Y]f``YUfb]b[ 'Yl dYf]YbW'`cZ`]a d`Ya Ybh]b[ 'h\Y]f`ck b'dYfgcbU` dfc'YWhUbX'di hUW\]Yj UV`Y'UWhcbg`]b'U'd`Ub'Zcf'h\Y'bYl h`YUfb]b[ 'qhU[Y"'H\Y'k cfX``]a ]h]g'&\$\$\$"

 $\label{eq:chinequality} Gh XYbhg fYWM]j Y'k f]hhYb ZYYXVUW_VUgYX'cb'fi Vf]Wg'''H\]g']g'h\Y'Z]bU''UggYgga YbhZcf'UhhYbXUbWM di fdcgYg''$ 

## FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

Gh Xmb[ ']b '9b[ `]g\. 'GhfUhY[ ]Yg 'Zcf'Gi WW gg ']b '< ][ \Yf'9 Xi WUh]cb 'f&\$%+Ł

#### FYWca a YbXYX FYgci fWfg

6cc\_'-'9ggYbh]U'g'cZ9ggUmK f]h]b[.'K \UhA Uf\_Yfg'@cc\_': cf'f&\$%+L

6cc\_'-'@YUfbYf'9b[\`]g\. '5'HYUW\Yff@'; i ]XY'hc'\bhYfZYfYbW''UbX'Ch\Yf'DfcV'Ya g'f&\$\$\L

6cc\_'-'7f]h]WU`FYUX]b[''UbX'K f]h]b[']b'h\Y'8][]HU``5[Y.'5b'\=bhfcXi Whcfm7ci fgYVcc\_'f&\$%\*Ł

6cc\_'-'@cb[a Ub'; fUa a Uf'cZGdc\_Yb'UbX'K f]thYb'9b[`]g\'ft\- ( \E'

6cc\_'-': cfY][b'@Ub[i U[Y'@YUfb]b['f%--, Ł

6cc\_'-'@YI]WJ``-ggi Yg']b'@Ub[i U[Y'@YUfb]b['f%-)Ł

6cc\_--<ck @Ub[i U[Yg'UfY'@YUfbYX'f&\$\$(Ł

6cc\_'-'; ]| Y'=1'U'; c.'HYUW\]b[ 'Dfcbi bW]Uh]cb'hc'5Xi 'hg'f&\$\$-Ł

6cc\_'-'HYUW\]b['UbX'FYqYUfW\]b['GdYU\_]b['f&\$%+Ł

A cfY fYgci fWg UfY fYWca a YbXYX cb @Y [ Ubhc f] ]U A ccXYŁ/ CdYb@YUfb]b[

#### 7ci fgY'9j U'i Uhcb'UbX'8Yj Y'cda Ybh

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhjb['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bz'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=I'mci
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbz'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\WJh]cb"'=Imci 'Ya U]''9! hYfbU''Gi ddcfhmci 'k]'`
Ui hca Uh]\WJ'`mfYW']j Y'U'h]\W\_Yhbi a VYf'ZVi h]Zmci 'hY'Yd\cbY'mci 'k]'`bYYX'hc'gdY\WZ\WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU``mž'UggYgga YbhiHug\_g'a i gh'VY'gi Va ]hhYX'Y`YVWfcb]\W)``mj ]\U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' \delta ]bgH\b\W'g'k \YfY'h\]g']g'bchdcgg]\V`Yž']hk ]```VY'gh\hYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYhJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5WUXYa]W±bZcfaUhjcb

: cf`YggYbh]U``gh`XYbh]bZcfa~Uh]cb`fY`Uh]b[~hc.

- fYei Yghg Zcf Yl hYbg]cb/`UhY gi Va ]gg]cbg [ i ]XY`]bYg/