

ARTS2384

Dc`]hJWU`D\]`cgcd\m

HYfa 'C bY''#'&\$%

7 ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
>Ua Yg [·] D\]``]dg	^d\]`]dg4 i bgk "YXi "Ui	A cbXUmg [*] &!' [*] da	A cfj Yb 6fck b	-',) &-,+

GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm!': f]XUmž'-Ua '!'(.() da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . 'Ž * %'&'-' ,) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY⁻8YHJ]`g

7 fYX]h'Dc]bhg '* '

GiaaUfmcZh\Y]7gcifgY

Gi VYWn5fYU. BZ[aeabZk

 $\label{thm:condition} $$H\Y'WcifgY'dfcj_XYg'mci'k_h'h'U'gc'XZcibXUhcb']b'dc'hWU'd\]`cgcd\mVmaYUbg'cZWcgY'fYUX]b[g'cZWbhU'hYlhg'Vmh\]b_Yfg'gi WX'Ug'H\caUg'<cVVYgž'>c\b'@cW_Y'UbX'<UbbU\'5fYbXh'Dc'h\W'd\]`cgcd\m$

5ggYgga Ybh

5ggYgga Ybh'Zcf'h\]g'Wci fgY']bj c`j Yg'h\fYY'UWUXYa]WYggUmg'cb'h\Y'h\fYY'gYhhYl hg"''K f]h]b[''Ub'UWUXYa]W YggUma YUbg'_YYd]b[']b'a]bX'gYj YfU`'HUg_g.

UŁ UWW fUHY mWcbj Ymjb['h\Y'a YUb]b['cZh\Y'df]a UfmhYl hfk \Uh']g'h\Y'hYl h U`'Yj]XYbW' Zcf'mci f lbhYfdfYhUhcb3ł/

VŁfYZYWIJb[cb h\ Y dcgg]V`Y k YU_bYggYg UbX]a d`]WUh]cbg cZh\ Y Ui h\ cf'g dcg]h]cbg fk \ Uhighf]_Yg mci 'Ug WcbhYbh]ci g`]b h\ Y fYUX]b['UbX#cf'k \ UhiZc``ck g Zfca ']h3Ł/

WE'g]hi Uh]b['mci f']bhYfdfYhUh]cb']b'fY`Uh]cb'hc'Yl]gh]b['Wca a YbhUf]Yg'fXcYg'mci f'hU_Y'cb'h\Y'hYl h'X]ZYf'Zfca k\Uhich\Yfg'\Uj Y'a UXY'cZ]h'UbX'WUb'mci 'dfcj]XY'fYUgcbg'Zcf'mci f'ck b'k UmcZfYUX]b['h\Y'hYl h'8Ł"

K]h\ci hfUŁ-'Ub'UWW fUhY'YI dcg]h]cb'cZh\Y'df]a UfmhYI h-'mci 'a U_Y']h\UfXYf'Zcf'mci f'fYUXYf'hc'gYYž'Zcf]bghUbWYž'h\Uh'mci f'Wf]h]WU`fYa Uf_g'ghf]_Y'\ca Y'fUbX'Xc'bch'\f gh`UbX'U'"_]``Yf'V`ck "'cb'U'Z][a YbhcZmci f]a U[]bUh]cbŁ"

K]h\ci hfl\t-'h\Y'YI dfYgg]cb'cZmci f'ck b'fYZYWf]cbg'cb'h\Y'df]a Ufm'hYI h-'mci 'a U_Y']h\UfXYf'Zcf'mci f fYUXYf'hc'gYY'h\Uh'gca Yh\]b[']g'VY]b['gU]X'h\Uh'\Ug'bch'VYYb'gU]X'VYZcfY"

K]h\ci hfWE-Yb[U[Ya Ybhk]h\ gYWbXUfm`]hYfUh fY'-mci 'a U_Y']h\UfXYf`Zcfmci f`fYUXYf`hc`gYY'\ck 'mci f]bhYfdfYhUh]cb`Wbhf]Vi hYg`rc`WbhYa dcfUfmUWUXYa]Wi bXYfgHUbX]b['cZh\Y']ggi Yg`UhghU_Yž'bY]h\Yf fYd`]WUh]b['k\Uhch\Yfg'\Uj Y`U`fYUXmgU]X`bcf`ZU``]b['Zci ``cZh\Y]f`Uf[i a Ybhg"

B6""? YYd]b[`fUłžffVł:`UbX`fWŁ]b`a]bX`]g`YI lfYa Y`miX]ZZ]W``hUbX`bYj Yf`VYWca Yg`glfU][\hzcfk UfX"``Hc`lfmž\ck Yj Yfž']g`lc`gYhci h]b`h\Y`f][\hiX]fYW]cb"

5g'U'fi 'Y'cZh\i a VžfYZYff]b['hc'Wca a YbhUhcfg'k \Yb'h\YmWf]h]W]gY'cf'Y`UVcfUhY'cb'h\Y'df]a UfmihYl hi]g VYhYf'h\Ub'fYZYff]b['hc'h\Ya'k \Yb'h\Ymg]a d'mdUfUd\fUgY']hcf'fYdYUhk]XY`m_bck b'ZJWg'WcbWYfb]b[']hg Wca dcg]h]cb'cf']bZi YbW''''=h]g'Yj Yb'VYhYf'k \Yb'mci 'WJb'di h'Zcfk UfX'Ub'Uf[i a YbhcZmci f'ck b']b fYgdcbgY'hc'U'Wca a YbhUhcf'g'Wf]hW]ga 'cf'Y`UVcfUh]cb'cZh\Y'df]a UfmihYl h'

5qqYqqa YbhFi Vf]W#9qqUmGhUbXUfXq

	< 8	8B	7 F	DG
91 dcg]h]cb cZ	7 cbj Ymgʻ]bʻU	7 cbj Ymgʻ]bʻU	7 cbj Ymgʻ]bʻU	7 cbj Ymg [·] U
]ggi Yg	₩:\YfYbha UbbYf'U	₩:\YfYbha UbbYf'U	₩:\YfYbha UbbYf'U	Wca dfY\Ybg]cb⁻cZ
	WYUf'UbX'dfcZcibX	WYUf	Wca dfY\Ybg]cb cZ	h\Y`]ggi Yg"
	Wca dfY\Ybg]cb cZ	Wca dfY\Ybg]cb cZ	h\Y`]ggi Yg"	
	h\Y`]ggi Yg"	h\Y`]ggi Yg"		
5bU`mg]g	91 \]V]hg^g_]``g	91 \]V]hg`g_]``g	91 \]V]hgʻgca Y	91 \]V]hgʻgca Y
	UggcW]UhYX^k]h\	UggcWjUhYX*k]h\	g_]``g`UggcW]UhYX	fYZYW1jcb"cb"h\Y
	h\Y`d\]`cgcd\]\W`	h\Y`d\]`cgcd\]\W`	k]h\^h\Y]ggi YgʻWcj YfYX"
	UbU`mg]gʻcZhYI hgž	UbU`mg]gʻcZhYI hgž	d\]`cgcd\]\W`	
	cZZYf]b[']bbcj Uhjj Y	g\ck]b[UbU`mg]gʻcZhYI hg"	
	UbX`]bg][\hil`	bXYdYbXYbWY ⁻ cZ		
]bhYfdfYhUh]cbg"	h\ci[\h"		
8]gWjd`]bUfm	8 Ya cbglfUllYg	8 Ya cbglfUllYg	: c``ckg	5 MmYa dhgʻhcʻi gYʻU
WcbjYbh]cbg	XYHJ]`YX`UHHYbH]cb	Wcbg]ghYbhii gY cZ	YI dYWMUNjcbg	Wcbg]ghYbh'gmghYa
	hc:UbXigiVWWggZi`]a dcfhUbh	Uddfcdf]UhY hc	Zcf [·] VUg]W

	k]XYfUb[YcZ McbjYbh]cbg dUfh]WUffch\Y UWUXYa]WYggUm]b d\]cgcd\mž]bWiX]b[cf[Ub]gUh]cbž	dUff]W`Uf'fc'f\Y UWUXYa]WYggUm]b d\]`cgcd\mž]bWiX]b[- 00	cf[Ub]gUh]cb UbX dfYgYbHJh]cb"
7]hUhjcbgʻUbX			8 Ya cbghfUhYg Ub UhhYa dhho i gY	8 Ya cbglfUllYgʻUb
eichUh]cbg	-	fYZYWNjYigYcZ fY`YjUbhgcifWYg	Ŭ	UmYadhhoʻigY gcifWygʻhoʻgiddofh
	gci fW/g hc	- C	hcigi ddofh]XYUgʻ]bʻh\YʻYggUmʻ
	UXj UbW Uf[i a Ybh'	dcg]h]cb"	dcg]h]cb"	
GmbHJI ž	gYg'[fUWYZ]`	l gYg	I gYg``Ub[i U[Y	I gYg``Ub[i U[Y
dibWhiUh]cb`UbX	`Ub[i U[Y`h\Uh	ghfU][\hZcfkUfX	h\Uh[YbYfU``m	h\Uhigca Yh]a Yg
jcWUVi`Ufm	Wcaaib]WUhYg]a dYXYgʻa YUb]b[
	a YUb]b[k]h\	, ,		VYWUigY'cZYffcfg
	WUf]miUbXiZi YbWn]b i gU[Y"
	UbX`]gʻj]fh U``m Yffcf! <i>]</i> fYY"	fYUXYfg"	kf]h]b[a Um]bWi XY gca Y	
	111011211	H\Y``Ub[i U[Y`]b	Yffcfg"	
		h\Y`YggUm\Ug`ZYk	3	
		Yffcfg"		

 $\label{eq:hamiltonian} $$HY^{*}]VfUfm\Ug^{*}UVWWgg^{*}hc^{*}h\Y^{*}7\]WU[\ c^{*}A\ Ubi\ U^{*}cZGhm^{*}Y.$

 $\label{thm:mkkkk} $$ \ k \ WX]WU[ca Ubi U`cZghm`Y"cf["kkkdfclm%"`]VfUfm'i bgk "YXi"Ui \#cc_\#X\%+\#fcbha UhhYf\#cW`\ha` `label{thm:mkkkk}.$

 $7\,cbgi\ `hih.]g'Zcf'XYhJ]'YX'UbX'Ui\ h.cf]hUh]j\ Y']bZcfa\ Uh]cb'cb'fYZYfYbW]b["$

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
FYgYUfVX YggUm%*\$\$ k cfXg	' \$ı		

GHJfhXUhY.

8YHJ]`g. `Ghi XYbhg`k f]hY`U`fYgYUfW\`YggUmcb`h\Y`Z]fghifYUX]b[`!`%*\$\$`k cfXg": YYXVUW_`k]```hU_Y`h\Y`Zcfa `cZU`fi Vf]WUbX`]bX]j]Xi U``Wca a Ybhg"

5XX]h]cbU`XYhU]`g.

: cWg. < cVVYg

9ggUmhcd]Wg.

%2'<cVVYg`dU]bhg`Ub`i bZUhhYf]b[`dcfhfU]hcZa Ub`]b`h\Y`ghUhY`cZbUh fY''`K \UhXcYg`h\]g`dcfhfU]h\Y`d <cVVYg`hc`Uf[i Y3``Hc`k\UhWf]h]W]ga g`XcYg`]hYl dcgY`\]g`UW\ci bhcZh\Y`Zci bX]b[`cZU`Wca a cbk YU`h\3

ʻŁ<cVVYgʻa UXYʻgYWʻf]mmh\YʻfUh]cbU`YʻcZh\Yʻa cXYfbʻghUhYʻUgʻZUfʻUgʻgi VʻYWgʻUfYʻWcbWYfbYX"∵∍gʻh\]gʻ\ck h\YʻghUhYʻ]gʻWcbWY]j YXʻhcXUm3∵5fYʻh\YfYʻU`hYfbUh]j Yg3∵K \UhdfcV`Ya gʻWUbʻmci ']XYbh]Zmk]h\ʻgYWʻf]hm UbX#cf`k]h\ʻU`hYfbUh]j YʻfUh]cbU`Ygʻgi W\ʻUgʻfY`][]ci gʻcVYX]YbWʻUbXʻh\YʻZci f]g\]b[ˈcZh\Y`]bX]j]Xi U`3

Ghi XYbhgʻUfYʻYbVci fU[YX'hc`XYj]gY'h\Y]fʻck bʻYggUmhcd]Vgʻzg]bW'ʻgY`Z\X]fYWYX`fYgYUfW\`UgʻUb i bXYf[fUXi UhYʻ]gʻUbʻYI WY``Ybhik UmhcʻdfYdUfYʻZcfʻdcgH[fUXi UhYʻghi Xm''D`YUgY`Wcbgi `hih\Y`Wci fgY Wcbj YbcfʻZcfʻUddfcj U``cZmci fʻW\cgYbʻhcd]VV

Hi fb]h]b gYh]b[. 'H\]g 'Ugg][ba Ybh]g 'gi Va]hhYX 'h\fci [\ 'Hi fb]h]b 'UbX 'gh XYbhg 'Xc 'bchgYY 'Hi fb]h]b g]a]'Uf]hmfYdcfhg"

5ggYgga Ybh& FYgYUfW YggUm%*\$\$ k cfXg

GHJfh:XUhY.

8YHJ]`g. 'Gh XYbhg`k f]hY`U`fYgYUfWk`YggUmcb`h\Y`gYWcbX`fYUX]b[`!`%*\$\$`k cfXg": YYXVUW_`k]```HU_Y`h\Y Zcfa 'cZU`fi Vf]WUbX`]bX]j]Xi U``Wca a Ybhg"

5XX]h]cbU`XYhJ]`g.

: cWg. @cW_Y

9ggUmhcd]Wg.

'E'@cW_Y'g'Uf[iaYbhiZcf'fY`][]cig'hc`YfUh]cb'XfUkg'cb'DfchYghUbhi]XYUg'WcbWYfb]b['h\Y'gUbWf]micZ]bX]j]XiU`'WcbgW]YbWY'''K\UhUfY']hg'ghfYb[h\g'UbX'kYU_bYggYg3

Gh XYbhg UfY YbWci fu[YX hc XYj]gY h Y]f ck b YggUmhcd]Wgzg]bWr gY ZX]fYWYX fYgYUfW Ug Ub

i bXYf[fUXi UhY`]g`Ub`Yl WY``Ybhik Umhc`dfYdUfY`Zcf`dcgh[fUXi UhY`ghi Xml`D`YUgY`Wcbgi `hih\Y`Wci fgY Wcbj Ybcf`Zcf`Uddfcj U``cZmci f`W\cgYb`hcd]W

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$

5ggYgga Ybh' . F YgYUfW . 9ggUm%* \$\$ k cfXg

GHJfh:XUhY.

8YHJ]`g. `Ghi XYbhg`k f]hY`U`fYgYUfW\`YggUmcb`h\Y`h\]fX`fYUX]b[`!`%*\$\$`k cfXg": YYXVUW_`k]```HJ_Y`h\Y`Zcfa cZU`fi Vf]WUbX`]bX]j]Xi U``Wca a Ybhg"H\]g`]g`h\Y`ZbU``UggYgga YbhZcf`UhYbXUbW\`di fdcgYg"

5XX]h]cbU`XYhU]`g.

: cWg. 5fYbXh

9ggUmhcd]Wg.

%5fYbXhik]ghg`UbX`gei YYnYg`\]ghcfmhc`Yl hfUWhZca `]hibYUhm\%cblfUgh]b[`h\Ycf]Yg`cZfYj c`i h]cb"``K \UhUfY`h\Y`VYbYZ]hg`UbX`XfUk VU_g`cZh\]g`UddfcU\\ hc`h\Y`\]ghcf]\\\

&£'5fYbXhfYh\]b_g'ZfYYXca ']b'hYfa g'cZdUff]W]dUh]cb']b'Wc``YWf]j Y'XYW]g]cb!a U_]b["''9j U`i UhY'\Yf'fYUgcbg Zcf'h\]g'fYXYZ|b]h]cb"

' Ł'5 fYbXhilfYUlg'h\Y'dc`]hJWD`'UWcf'Ug'U'g\cfh`]j YX'd\Ybca Ybcb'cZh\Y'a cXYfb'dYf]cX''''K \UhZUWcfg'XcYg g\Y'WU]a 'Wcblf]Vi hY'hc'h\Y'XYa]gY'cZh\Y'Yi d\cf]WZfYYXca 'cZdc`]hJWD`'UWJcb3''K \Uh]g'mci f'UggYgga Ybh cZ\Yf'dcg]hJcb3

Ghi XYbhgʻUfYʻYbWci fU[YX'hcʻXYj]gY'h\Y]fʻck bʻYggUmhcd]Wgʻzg]bWʻgY`Z\X]fYWYX`fYgYUfW.`UgʻUb i bXYf[fUXi UhYʻ]gʻUbʻYI WY``Ybhk UmhcʻdfYdUfYʻZcfʻdcgh[fUXi UhYʻghi Xm''D`YUgY`Wcbgi `hh\Y`Wci fgY Wcbj YbcfʻZcfʻUddfcj U``cZmci fʻWkcgYbʻhcd]W

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh"]g"gi Va]hhYX"h\]fci [\" 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{array}$

5ggYgga Ybh(. 'A ccX`Y 'Dcghg

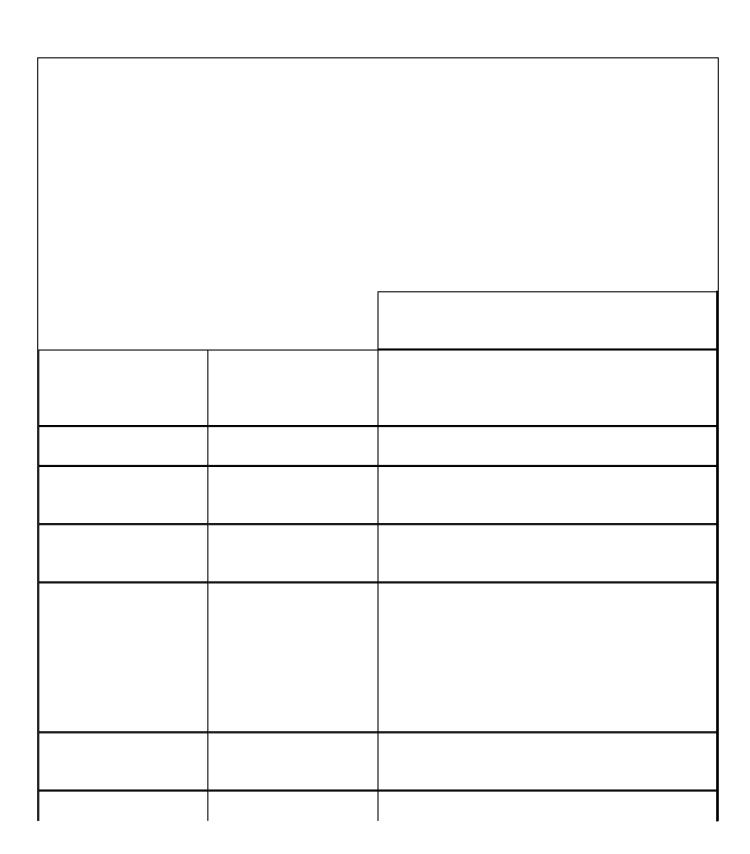
GhUfh XUhY.

8YHJ]`g. `Ghi XYbhg`dcghicb`A ccX`Y'' L %\$\$!%) \$`k cfXgž'cbY`dcghiZcf`YUW\`cZh\Y`h\fYY`gYhhYI hgž']b fYgdcbgY`hc`X]gW gg]cb`hcd]Wg`XfUk b`i d`Vmghi XYbhg`YUW\`k YY_`Xi f]b[`h\Y`h hcf]U`g"7 cbj Ybcf`dfcj]XYg ZYXVUW_`VmfYgdcbX]b[`hc`h\Y`A ccX`Y`dcghg"

5XX]h]cbU`XYhJ]`g.

Ghi XYbhgʻUfYʻfYei]fYXʻhcʻdcghiUʻhchUʻʻcZh\fYYʻWbhf]Vi h]cbgʻ—ʻcbYʻcbʻYUW\ʻcZh\Yʻh\fYYʻVcc_gʻk Yʻk]```VY fYUX]b[`—'hc`h\Y`X]gW gg]cbʻZcfUʻcbʻA ccX`YʻVmih\Y`YbX`cZh\Y`gYa YghYf"

Hcd]\Mg'Zcf'th\Y'X]g\W gg]cb'Zcfi a 'k]``\VY'XfUk b'i d'YU\W\'k YY_`]b'th\Y'ti tcf]U``UbX'dcgt\X'\Vmth\Y'\\chi fgY\\chib\) Ybcf"'\\frac{1}{3}i bk]g\Y'tc``\YUj \Y']hi bt]``th\Y'\YbX'cZth\Y'g\Ya \Ygth\frac{1}{3}b'tc``\Y\[]b'dcgt\b["



A Um		FYUX]b[.'5fYbXhžA` DWha^gf[a`ždd"'(-!%%	
K YY_"%% "%("A Um'! "&\$	@YWhi fY	Hcd]W'H\Y'5a Yf]WUb'F Yj c'i h]cb	
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K YY_'%' . '&, 'A Um'! ''	Hi hcf]U`	Hi hcf]U``cb`m''`Bc``YWh fY"	
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%" H\ca Ug`< cVVYgž'>Wh[SfZS` žYX"`7 "`6"`A UWd\Yfgcb`f@cbXcb. `DYb[i]bž'\%,) \text{L"} &" >c\b`@c\\Yz`FZ\WE\\Ua`\V\Fd\\Sf[e\WaX9\ah\\d_\Wf\S`\V\3\>\Wf\\d\5\a`\U\\d\[`\Y\Fa\\\d\Sf[a`\`fA\]b\Yc`\Uz\\B\M`8\cj\Yfz`\&\\$\&\text{L"}

' " < UbbU\ '5 fYbXhž'A \ D Wha 'gf[a \ 'f@cbXcb.' DYb[i]bž'&\$\$*Ł"

FYWca a YbXYX FYgci fW/g

Gi [[YghYX Z fh\YffYUX]b[cb < cVVYg.

 $5i\ VfYm\check{z}>c\b"\text{``H}\c a\ Ug'<cVVYg\text{'`'}]b\text{`}4\text{ q(WK}>[hWe\check{z}\text{'H}\YfY\text{`}UfY\text{'}a\ i\ \text{`h]d}\text{`Y}\text{'}YX]h]cbg\text{`}cZh\text{`}]g\text{``WY}\text{`YV}fUhYX\text{'}YUfm\text{'}V]c[fUd\m']$

6Ui a [c`Xž'8YVcfU\"'&\$\$, "'"H\Y'8]Z]W\`h]Yg`cZ<cVVYg`=bHYfdfYhUh]cb""'Ba^{f[US^FZWack'*.', &+!))"

Gcj YfY][b'UbX'DYcd`Y']b'<cVVYg'g'Dc`]f]WU``H\Ycfm'''' $4 df[eZ < agd S^$ XadfZW: [efack aXBZ['aeabZk % . ') +) ! - *"

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6Y^Ubž'H""A ""&\$%* """@cW_Y "cb"Hc`YfUh]cbž'f\\b\z']]]m\UbX'h\Y'E i Ygh\Zcf'7 cb\\cfX"\": [efack

aXBa/ff[US^FZagYZf' +. ')) *!, +"

6fUXmžA]W\Y``Y"`&\$%+"`~5Wnjb[`Zcf'h\Y`DiV`]W; ccX.`@cW_Y`cb': fYYXca `UbX'>i X[a Ybh'~;` fWd Sf[a`S^BZ['aeabZ[US^CgSdrWdk')+.'('!*\$"

6fck bž'J]j]YbbY"'% - - "'"H\Y':][i fY''cZ; cX'UbX'h\Y'@]a]hg'hc'@]VYfU`]ga .'5'''''F YfYUX]b['cZ@cW_Y'g '9ggUm''UbX''Hk c'HfYUh]gYg'""'<agd S^aXfZW: [efack aX ;VWSe *\$.', '!%\$\$"

; fUbhžFih\`K "`&\$%&"`">c\b`@cW_Y`cb`7ighca 'g`DckYf`UbX`FYUgcb'g`5ih\cf]hm'"```FZW DWh[W aX Ba'ff[Ue'+(.`*\$+!&-"

A cchgž; 'Ybb'UbX'; fY[': cfghYf"'&\$%\$"'"ES'ge babg'[egbdW_S 'W'].'>c\b'@cW_Y'j Yfgi g'7cbhYa dcfUfm 8Ya cWfUhWH\Ycfm'"BWdebWUfjhWe a` Ba'[ffUS^EU[W UW' -.'')!()"

A mYfgž'DYhYf'7"'% -) "'"6Yhk YYb'8]j]bY'UbX'< i a Ub'Gcj YfY][bhm'H\Y'GhUhY'cZBUhi fY'UbX'h\Y'6Ug]g'cZ @c \underline{W} Y'g'Dc`]h] \underline{W} \'H\ci [\h'"'Ba'{fk'8}+\\"*&+\\"*

DUgei]bcžDUgei U`Y"'% - , "'"@c \underline{W} Y'cb'?]b['g'DfYfc[Uhjj Y""'Ba'{f[US^FZWadk, 3` ;`fWd Sf[a`S^<agd S^aXBa'ff[US^Zkž

 $EaU[S^5d[f[U[e_ "]]] - **%] - **$

 $\label{eq:hydro} H]YfbYm\H{z}`6f]Ub"`\&\$\$)"""<]ghcf]WU``Fcchg`cZ`AcXYfb`F][\hg.`6YZcfY`@cW_Y`UbX`5ZhYf""`3hW?Sd[S>SiDWh[W''.8'!('")]$

j Ub 'XYf'GWXUUfž'A Uf]U" '&\$\%\"'"@c_Y'cb'>i X[Ya Ybh'UbX'FY`][]ci g'Hc`YfUh]cb"\"'4\df[eZ <agd S^\\adfZW : [efack aXBZ[\aeabZk'&\\$.'(\%*, "

 $\label{eq:continuity} $J \in [h\check{z} D]^{-1} d''' - +"```@c W_Y \check{z} = XYb ``UbX ``Hk c ``GHUHYg ``c ZB UHi fY . ``H \Y `: cfhi bUHY `: U```````F Yj]g]HYX'''' < agd S^aX fZW: [efack aXBZ['aeabZk'') . ``) &' !(("$

 $K U Xfcb z' > YfYa m'' % , - "`" > c b' @c W_Y . `Gc W]U ``7 cb ff U Whij Yfgi g' Dc `]h] W U ``5 bh fcdc `c [m''' `F Z W D Wh [W aX Ba 4 ff [Ue') % '' ! & , "$

"'&\$\$&"'9aV 1 >aU] WS` V 7cgS'[fk, 5Zd[ef[S` 8ag` VSf[a` e aX>aU] We Ba'[f[US^FZagYZf#

A U[i bž5ffYa m'i &\$\$+" "H\Y '8 ci V'Y '6]bX. 'H\Y '5a V]j U'Ybh'HfYUha Ybh'cZHfU[]WDUgg]cbg ']b '< UbbU\ 5fYbXhg'H\YcfmcZF Yj c'i h]cb"": [efack aXBa'{f[US^FZagYZf &, .'+%! (* "

J]``Už'8 UbU'F]W\UfX"'%--"'Ba'{f[Ueł BZ['aeabZkł FWddad, 7eeSke a` fZWFZagYZfaX: S``SZ3dW VfZDf]bWhcbzB>.'Df]bWhcb'l b]j Yfg]hmDfYgg"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

- VYHYfa UbU[Ymci fha Y
- i bXYfglUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK
- VY 'Uk UfY 'cZd`U[]Uf]ga ž Wcdmf][\hž·I BGK 'Ghi XYbhi7cXY 'cZ7cbXi WhUbX '5WWdhUV`Y 'I gY 'cZ I BGK '=7 H'F Ygci fWYg'Dc`]Wh
- VY'Uk UfY'cZh\Y'ghUbXUfXg'cZVY\Uj]cif'YldYWhYX'cZYjYfmcbY']b'h\Y'l BGK 'Wcaaib]hm
- `cWUhY gYfj]WYg UbX]bZcfa Uh]cb UVci hI BGK 'UbX I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``'VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``'VY'bYk"'''

K =7 H°F Ygci fWYg°Dc`]Wm

5 WUXYa]W = b Zcfa Uh cb.

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj]Yk `cZa Uf_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm'Yl Ua]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/ghi XYbhYei]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa]W]bZcfa Uh]cbž gYY`\hdg.#k k k "Ufhq"i bqk "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa]W]bZcfa Uh]cb#dfchcWc`q![i]XY`]bYq#

a U[Y'7fYX]h

GmbYf[]Yg^]b^GcibX^&\$%*

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;