

# **ARTS3751**

8 Yj Y`cda Ybh'Gh X]Yg. F YgYUfW\ž'DfUWf]WY 'UbX '9 h\]Wg

HYfa 'H\fYY''##'&\$%-

### 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \*

Gi a a UfmcZh\Y'7ci fgY

Gi VYWh5fYU. 6WhWab\_WfEfgV[We

#### 7ci fgY @YUfb]b[ Ci hWca Yg

%" 9 I d`U]b`h\Y`fY`Uh]cbg\]dg`VYhk YYb`fYgYUfW\ž'dc`]Whž'UbX`U[ YbWhi]b`XYj Y`cda Ybh &" 7 f]h]WU``mYj U`i UhY`h\Y`k Umg`XYj Y`cda YbhU[ YbW]Yg`cdYfUhY`UbX`h\Y]f`]a dUWg ' " `7 cbXi Whi]bXYdYbXYbhUbX#cf`Wc``UVcfUh]j Y`fYgYUfW\`]bhc`XYj Y`cda YbhdfUW]WY`UbX`Yh\]Wg ( " `5 dd`mWcbWYdhg`UbX`hcc`g`XYj Y`cdYX`]b`h\Y`Wci fgY`hc`gWYbUf]cg`UbX`WUgY`ghi X]Yg

#### HYUWX]b[ 'GhfUhY[]Yq'

H\Y''!\cif`gYa]bUf`]g`XYg][bYX'bc`fYZYWbh\Y'mdY`cZfYgYUfW\žkf]b]b[ž'UbX`]b!XYdh\\_bck`YX[Y`fYei]fYX]b`dfcZygg]cbU``gYhdb[g`cZXYjY`cdaYbhdfUWfW"'=i]g`U`gc`XYg][bYX'bc'YbWcifU[Y'h\Y'\_]bXg`cZWc``UVcfUhjY'k cf\_'UbX'dYYf`ZYYXVUW\_h\Uh\UhY'Wcaacb']b'giW\dfcZygg]cbU``gYhdb[g"

Mci 'gdYbX'h\Y'gYa YghYf']b'K cf\_]b['; fci dg'UbX'k]```ghUm']b'h\YgY'Zcf'h\Y'gYa YghYf'''BchU```cZh\Y gYa]bUf'X]gW gg]cbg'k]``]bj c'j Y'[fci d'k cf\_ž'Vi hUh'YUghgca Y'cZh\Y'h]a Y'YUW\k YY\_k]``''

## 5ggYgga Ybh

- 5ggYgga Ybhgʻg\ci`X`VY`fYZYfYbWYX`]b`UVW¢fXUbWY`k]h\`h\Y`GW\cc``cZGcV]U``GV]YbWYg FYZYfYbW]b['; i]XYž'UjU]`UV`Y`Uh \hhdg.##gcW]U`gV]YbWYg"Ufhg"i bgk "YXi"Ui #a YX]U#GCGG:]`Y#GcGGSFYZYfYbW]b[S; i]XYS&\$% "dXZ
- BcZcchbch/gždYUgY\*\*H\Y\*GcGG\*[i]XY\*igYg\*Ub\*]b!h/IhWjhUhjcb\*gmgh/a "
- D`YUgY`Xc`BCH`i gY`kYV`fYZYfYbWY`dU[Yg`gi WX`Ug`K]\_]dYX]U`YhW'CZZ]W]U``U]X cf[Ub]gUh]cb`kYVg]hYg`UfY`C?`hc`i gY"

Ghi XYbhgʻk ]```fYWY]j YʻUbʻ]bhfcXi Whcbʻhcʻh\]gʻUggYgga Ybhi]bʻK YY\_'%UbXʻh\YʻWti fgYʻWtbhYbhi]bʻK YY\_'%UbX &'k ]```\Uj Yʻ[ fci dʻgYgg]cbgʻcbʻh\]gʻUggYgga Ybhhcʻdfcj ]XYʻghi XYbhgʻk ]h\`dYYfʻZYYXVUW\_"``: cfʻa cfYʻXYhU]`g Wtbqi `hA ccX`Y"`

Gi Va ]gg]cb bchYg.Cb ]bY gi Va ]gg]cb cb A ccXY

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYh]b[. Hi]g Ugg][ba Ybh]g gi Va]hYX h fci [\ 'Hi fb]h]b UbX ghi XYbhg WUb gYY 'Hi fb]h]b g]a] Uf]hm fYdcfhg"$ 

5qqYgga Ybh& 7f]h]WU`Dc`]Wm6`c[

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\., \$\$!-\$\$ k cfXg

8 YHJ]`g. `Ghi XYbhg`]XYbhjZmU'dfYgg]b[ `dfcV`Ya `]b`]bhYfbUhjcbU``XYj Y`cda Ybh'UbX`XYj Y`cd'Ub`UbU`mhj\WU`V`c[ `cZYf]b[ `dc`]\Wngc`i hjcbg"'H\Y'V`c[ `g\ci `X'VY'Ufci bX', \$\$!-\$\$`k cfXg"'Ghi XYbhg`k ]```VY`UggYggYX VUgYX`cb`U'fi Vf]\Wa UXY`Uj U]`UV`Y`hc`h\Ya 'Uhh\Y'ghUfhcZh\Y'\Wci fgY`gc`h\Ym\Wb`k cf\_'hck UfXg`gdY\\Z]YX ghUbXUfXg"'K f]\mathred{h}\b'zYYXVU\W\_'UbX`U'bi a Yf]\\W\`[ fUXY`k ]```VY`dfcj ]\XYX`k ]\mathred{h}\]b'hYb'k cf\_]b[ `XUmg`cZ gi Va ]gg]cb"``

#### 5XX]h]cbU`XYhU]`g.

 $D`YUgY`Wcbgi `hA ccX`Y`Zcf`Z ff\ Yf`]bZcfa Uh]cb"``Ghi XYbhg`k ]```VY`dfcj ]XYX`k ]h\ `hcc`g`cb`\ck `hc`k f]hY`V`c[g UbX`cbY`GYa ]bUf`k ]```Wcj Yf`_Ymg_]``g"``$ 

Gi Va ]gg]cb bchYg.Cb`]bY A ccX`Y UggYgga Ybh

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYh]b[."H\]g Ugg][ba Ybh]g gi Va]hhYX"h\fci [\h]h]b UbX"ghi XYbhg Wbb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \\ \end{tabular}$ 

5qqYqqa Ybh' DcfhZc`]c

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.' ž\$\$\$ k cfXg a Ul]a i a

#### 5XX]h]cbU`XYhJ]`g.

D`YUgY`Wcbgi `hA ccX`Y`Zcf`Z fth\Yf`]bZcfa Ut]cb"``H\]g`dcfttZc`]c`]g`th\Y`W`a i `Ut]cb`cZU`gti XYbtfig`]bXYdYbXYbh fYgYUfW. WcbXi WtX`Xi f]b[ 'th\Y`Wci fgY`U]a ]b[ 'UtnZ]``]b[ '[ Udg`]XYbt]Z]YX`]b`th\Y`]b]tjU``UggYgga Ybti'th\]g a ][ \thVY`cbY``cb[ Yf`d]YWY`ZfYgYUfW. 'cf`U`Wci d`Y`cZg\cftYf`cbYg"`H\Y`WcbtYbti]g`tc`VY`U[ fYYX`k ]th\th\Y Wci fgY`Wcbj YbYf`]XYU``mVmK YY\_'("``C bY`dUffith\UtiU```gti XYbtg`\Uj Y`tc`]bWi XY`]b`th\Y`dcfttzc`]c`]g`U fYZYWf]cb`cb``YUfb]b[ g`]b`th\Y`Wci fgY`UbX`cj YfU```gtfYUa "``

Gi Va ]gg]cb bchYg.Cb`]bY A ccX`Y gi Va ]gg]cb

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b"[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$ 

## 5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj ]Yk '`YWhi fY`fYWcfX]b[g"

# 7ci fgY'GWXYXi `Y

#### J]Yk 'WUgg'ha YhUV'Y

### H]a YtUV`Y

| 8 UHY                                      | Hmd Y           | 7 cbhYbh   |
|--|-----------------|--|
| K YY_'%'%'GYdhYa VYf'!<br>&\$'GYdhYa VYf   | @YWn fY         | #olfcXi Wgcb"hc"h\Y"8Yj Y`cda YbhGhi X]Yg"7UdghcbY<br>!; Yhgb["hc"_bck "YUVX"ch\Yf"  |
|  |                 | ±blfcXi Wjcb'hc'GK CH'UbU'mg]g   |
|  | Cb`]bY'5Wfjj]fm | F Yj ]Yk '&!' 'cZh\Y'cb']bY'gYWcf''cV'g]hYg'dfcj ]XYX'rcmci 'cb'A ccX'Y"''GY'YWiUVci hih\fYY!gYWcf''cV UXj Yfh]gYa Ybhg'h\Uhmci 'k ci 'X'VY']bhYfYghYX']b Udd'm]b[ 'Zcf'f]XYU'`mZfca 'X]ZZYfYbhU[ YbW]YgŁ'UbX a U_Y'U'`]ghicZWca a cb'g_]``g#_bck `YX[ Y'fYei ]fYX" C b'a ccX'YžYUW\'ghi XYbhk ]``VY'[ ]j Yb'Ub'cb`]bY gdUW' 'hc'dcghih\Y'cV'UXg'Ug'k Y``'Ug'h\Y]f g_]``#_bck `YX[ Y'gi a a Ufm'''H\]g'cb`]bY'UWhj ]hm'Zcfa g h\Y'VUg]g'Zcf'[ fci d'X]gW gg]cb']b'K YY_'&'UbX'H\]g UggYgga Ybhk ]```\Y'd'mci 'k ]h\'h\Y'[ Ud'UbU'mg]g'h\Uh |
|  |                 | Zcfa g h\Y Zci bXUh]cb Zcf UggYgga Ybh%"   |
| K YY_`&.`&' `GYdhYa VYf`!<br>&+`GYdhYa VYf | GYa ]bUf        | H\Y'GYa]bUf'k]``ZcW g'cb'[fci d'k cf_'k \YfY'mci WcbXi Wh'U'[Ud'UbU'mg]g'VUgYX'cb'h\Y''cV UXj Yfh]gYa Ybhg'Wcbgi `hYX'UbX'ck b'g_]``g'UggYggYX VUgYX'cb'_bck `YX[Y'UWei]fYX'Xi f]b['h\Y]f i b]j Yfg]hmghi X]Yg"'5hh\Y'YbX'cZh\Y'GYa]bUfž ghi XYbhg']XYbh]ZmU'_Ym[Ud'h\Ymk Ubhhc']a dfcj Y i dcb'Xi f]b['h\Y'7 UdghcbY"'  |
| K YY_" ." \$"GYdhYa VYf"!                  | @YWhi fY        | =blfcXi Wgcb"tc"FYZYWgjY"DfUWgW"   |
| (°CWcVYf                                   | Cb`]bY`5Wfj]lm  | FYUX 'Un'YUghH < F99'X]ZZYfYbhV'c[g']b  +bhYfbUhJcbU''8YjY'cda Ybh'fZfca 'gi [[YghYX'g]hYgŁ UbX']XYbhJZmcbY'cf'kk c'Wf]hYf]U'cZU'f[iccXfiV'c['UbX cbY'cf'kk c'Wf]hYf]U'cZU'fVUX'V'c[f''Ghi XYbhg'k]''VY dfcj ]XYX'k]h\'Ub'cb']bY'GdUWY'cb'A ccX'Y'hc'g\UfY cbY'f[iccXfiUbX'cbY'fVUXfV'c['UbX'h\Y']XYbhJZYX Wf]hYf]U"''H\YgY'k]''Zcfa 'h\Y'VUg]g'Zcf'X]gW gg]cbg']b [fci d'kcf_Xi f]b['KYY_'("''  |
| K YY_'(.'+'CWhcVYf'!'%%<br>CWhcVYf         | GYa ]bUf        | 7 cblf]Vi ljb[ 'lc '8 Y V U lyg ']b ' + b lyfb U ljc b U ` 8 Yj Y`cda Y b n # - a d U lyfb[ ' + b lyfb U ljc b U ` 8 Yj Y`cda Y b n D c`] lym # + b lfc X i lyfjc b 'cb 'lc '\ck 'lc k f]ly ' g\cfn V`c[ g#dc`] lym V f] Y Zg" 'H \]g 'G Y a ] b U f`]g 'l\ Y Zci b W U ljc b 'Zc f' l\ Y 'g Y W cb X 'U gg Y gga Y b n U b X ']b j c`j Y g U 'a ] I h f Y 'c Z Y W f Y Z Y I Y f W g Y g' U b X '[ fci d k cf_"'  |
|  | 5ggYgga Ybh     | 5ggYgga Ybh'%'! '5bU'mh]WU' '9ggUmf&) ı ž', \$\$!-\$\$<br>k cfXgŁ'Xi Y'VYZcfY' '&&!bccb'cb'- 'C WcVYf''  |

|  | K YY_').'%('CW)cVYf'!'%,<br>CW)cVYf           | GYa ]bUf | 9h\]Wg`]b`=bHYfbUH]cbU``8YjY`cdaYbh#8c'bc'\Ufa"'                                     |
|--|---|----------|--|
|  | K YY_'*.'&%'CWhcVYf'!'&)<br>CWhcVYf           | GYa ]bUf | ; Yhhjb[ 'fYUXmZcf'h\Y'&V'a Uf_Yki! '7 UfYYfg']b folk<br>+bhYfbUhjcbU`'8Yj Y`cda Ybh |
|  |   |          | K f]h]b[ 'U'gi WWggZ `'&V'Udd`]WUh]cb'#7J 'fh\YfY'k]``<br>U`gc'VY'Ub'cb`]bY'UWfj]]mŁ |
| K YY_'+.'&, 'C W<br>h\Y'gYW <b>ndB'#_EYØDXY5</b> | K YY_'+.'&, 'CWcVYf'!'%<br>B <b>#£Y∀bXY</b> 5 | GYa ]bUf | ±bXi ghfm#5`i a b]'DUbY`'!'k \Uh'XcYg']h'hU_Y'hc'k cf_']b<br>h\Y'gYWcf'#E 'UbX'5     |
|  |   |          |  |
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|  |   |          |  |
|  |   |          |  |

## Fygci fWyg

### DfYgWf]VYX F Ygci fWYg

 $5 \text{ `fYUX]b[ ``]ghcZWca di `gcfmfYUX]b[ g'Uj U]`UV`Y \text{'hhfci [ \ 'hhY'I BGK ``]VfUfmcb`]bY']g'Uj U]`UV`Y'cb'ci f A ccX`Y'dU[ Y'ZcfYUW\'k YY_"''$ 

### FYWca a YbXYX FYgci fWYg

D`YUgY`Wcbgi `hA ccX`Y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

## 5 WUXYa ]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb[ 'hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va ]gg]cbgʻ[ i ]XY`]bYg/ fYj ]Yk ʻcZa Uf\_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/