

# **ARTS3759**

DfYdUf]b[ 'Zcf'UbX'F YZYWf]b[ 'cb' +bhYfbUh]cbU` +bhYfbg\]dg

HYfa 'C bY''# '&\$&\$

# 7 ci fgY Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g`

#### '7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
Gi gUbbY GW\a Y]X`	g"gVXa Y]X`4 i bgk "YXi "Ui	Vm'Uddc]bha Ybh	%, A cfj Yb	f\$&Ł'-',)
			6fck b 6i ] X]b[	&(\$)

## GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

### 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \*

#### Gi a a UfmcZh\Y'7ci fgY

Gi VYWn5fYU. 6WhWab\_W f EfgV[We

H\]g`Wci fgY`dfYdUfYg`mci `Zcf`h\Y`]bhYbg]j Y`XYa UbXg`cZUb`]bhYfbUhJcbU``k cf\_!]bhY[fUhYX``YUfb]b[ Yl dYf]YbWY"'=b`h\Y`Zci f`XUmg`cZ]bhYbg]j Y`k cf\_g\cd`UWhj ]hJYgž`mci `k ]``X]gW gg`h\Y`]a dcffUbWY`cZ WcbhYl hi U`]g]b[`mci f`Yl dYf]YbWYž'UbX`XYj Y`cd`fYZYWhj Y`g\_]``g`bYWYggUfmhc`a Ul ]a ]gY`mci f``YUfb]b[`cb d`UWYa Ybh''5`][ bYX`k ]h\`l BGK 'g`Ya d\Ug]g`cb`k cf\_!]bhY[fUhYX``YUfb]b[žh\]g`Wci fgY`]g`XYg][ bYX`hc Ybgi fY`h\Uhmci `[Yhh\Y`a cghci hcZmci f`]bhYfbg\]d`cf`d`UWYa Ybhcj YfgYUgž'UbX`h\Uhmci `WUb`Vch\ Wcbhf]Vi hY`hcž'UbX`VYbYZhZca `mci f`dYf]cX`k ]h\žmci f`\cghcf[ Ub]gUhJcb`]b`Ub`Yh\]WU`žfYgdYWMZ `ž'UbX a i hi U``mdfcXi Whj Y`k Um'

#### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

- %" 5fhW 'UhY 'Ub'i bXYfghUbX]b[ 'cZ\ck 'h\Ycf]Yg'g\UdY'dfUWNW"'
- &" 91\]V]hWf]hJWU``Uk UfYbYgg`UbX`WcbhYl HgYbg]hjj Y`UddfYWJUhJcb`cZXYj Y`cda Ybhž`\i a Ub]hUf]Ubž UbX#cf`Wca a i b]hmdfUWfW"
- ' " 9j U`i UhY`h\Y`Yh\]WU``X]a Ybg]cbg`cZXYj Y`cda Ybhz`\i a Ub]hUf]Ubz`UbX#cf`Wca a i b]hmidfUWjWYz ]bWi X]b[ `Vi hbch`]a ]hYX`hc`\i a Ub`f][ \hg`UbX`gcWjU``^ ghjWY`df]bWjd`Yg"`
- ("FYZYWicb'UbX'YjU'iUhY'kcfd'UWY'dfUWf)WYz'dfcZYgg]cbU``XYjY`cdaYbhz'UbX``YUfb]b["

#### HYUWX]b[ 'GhfUhY[]Yq'

H\]g`Wci fgY`i gYg`DfcV`Ya!6UgYX`@YUfb]b[`fD6@Łk\]WX`di hg`h\Y`ghi XYbhUhh\Y`WYbhY`cZh\Y``YUfb]b[ ci fbYmžfYei]f]b[`h\Ya`hc`d`UmUb`UWf] Y`fc`Y`]b`h\Y``YUfb]b[`dfcWYgg"`Ghi XYbhg`UfY`Yb[U[YX`k]h\ Wcbhf]Vi h]b[`hc`fYU`!`]ZY``dfcV`Ya g"`H\]g`a YUbg`h\Uhighi XYbhg`UfY`UV`Y`hc`Udd`mik\Uhih\Ym`YUfbYXž'UWei]fY bYk`g\_]``g`fZ]Y`X`\_bck`YX[Yž'hYUa`k cf\_žcf[Ub]gUh]cbU``\_bck`YX[YŁ'h\Uhik]``VYbYZ]hih\Y]f`Zi hi fY WUfYYfg"H\YfY`UfY`gY] YfU``fYUgcbg`Zcf`i g]b[`D6@`]b`h\]g`Wci fgY.`

#ifYj c'j Yg'Ufci bX'h\Y'gh XYbh' #D'U'D6@Ybj]fcba Ybhgh XYbhg'UfY'dYfgcbU``mfYgdcbg]V`Y'Zcf'k\Uh h\Ym'YUfb"'H\YmHU\_Y'dUfhi]b'X]gW gg]cbgžg\UfY'\_bck `YX[Y'UbXž'hc[Yh\Yf'k]h\'h\Y]f'\cghcf[Ub]nUh]cb Zcfa i `UhY'mci f'ck b'`YUfb]b['cVYWh] Yq"'

5 Wei ]f]b[ 'g\_]``g'Zcf``]ZY. '5 b']bhYbg]j Y'k cf\_!Yl dYf]YbW' WUb'dfcj ]XY'ghi XYbhg'k ]h\ 'h\ Y'Z]fghiHughY'cZk \Uh']h a YUbg'hc'VY'U'XYj Y`cda YbhidfcZygg]cbU``UbX'[ U]b'g\_]``g'h\YmUfY'i b`]\_Y`mhc'UWei ]fY']b'U'WUgg!fcca gYhjb['"5 Z]Y`X']bhYfbg\]d']b'U'W' hi fU``mX]j YfgY'Ybj ]fcba Ybha U\_]b[ 'h\Y'X]gW gg]cbg'YgdYW]U``m`]j Y`mUbX VY]b[ 'dUfhcZU'hYUa ']b'U'Wca a i b]hmcf[ Ub]nUh]cb'dfYgYbhg'ghi XYbhik ]h\'h\Y'cddcfhi b]hmhc'k cf\_ik ]h\ dYcd'Y'Zfca '\][ \`mX]j Yf[ YbhX]gW]d`]bUfmVUW\_[ fci bXg"'Ghi XYbhg 'YUfb'YggYbh]U`'g\_]``gžgi W\'Ug``]ghYb]b[ hc'ch\Yfgz'dfYgYbh]b[ 'h\Y]f'j ]Yk dc|bhz'XYVUh]b[ ž'k f]h]b[ 'fYdcfhg'UbX'k cf\_]b[ 'Wc``UVcfUh]j Y`m'

# 5ggYgga Ybh

<ul> <li>5ggYgga Ybhgʻg\ci `X'VY`fYZYfYbWYX`]b`UVW\cfXUbWY`k]h\`h\Y`GVXcc``cZGcWJU``GWJYbWYg FYZYfYbWJb[`; i]XYZ'Uj U]`UV`Y`Uh`</li> </ul>							
\	hhdg.##gcWJU`gWJYbWYg"Uf	hg"i bgk "YX	(i "Ui #a Y	X]U#GC GG:	]`Y#GcGGSF	YZYfYbW]b[S;	i ]XYS&\$% "dXZ

#### 5XX]h]cbU`XYhJ]`g.

D`YUgY`Wcbgi `h'A ccX`Y`Zcf`Z fh\Yf`]bZcfa Uhjcb

Gi Va ]gg]cb bchYg.Cb ]bY gi Va ]gg]cb

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYh]b[. Hi]g Ugg][ba Ybh]g gi Va]hYX h fci [\ 'Hi fb]h]b UbX ghi XYbhg WUb gYY 'Hi fb]h]b g]a] Uf]hm fYdcfhg"$ 

5ggYgga Ybh& DfY!8YdUfh fY DcfhZc`]c

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'%ž)\$\$

8 YHJ]`g. `Ghi XYbhg`gi Va ]hU'h\fYY!dUfndcfhzc`]c`flvz) \$\$ 'k cfXgŁ'k \]W\ 'Wcbg]ghg`cZg\cfh'V`c[g`ci h]b]b[h\Y]fUŁ'Dcg]h]cbU`]hmVŁ'±bhYfbg\]d`9| dYWfUh]cbgUbX`WŁ'F]g\_'UggYgga Ybh#a]b]a]gUh]cb`ghfUhY[m'Ghi XYbhg`k]``VY`UggYggYX`VUgYX`cb`U'fi Vf]WaUXY`Uj U]`UV`Y`hc`h\Ya`Uhh\Y`gfUfncZh\Y`Wci fgY`gc`h\YmWb`k cf\_

#### 5 hhyb XUb WY F Yei ]fYa Ybhg

H\]g`Wci fgY`]g`cZYfYX`]b`V`YbXYX`a cXi g"``Ghi XYbhg`UfY`fYei ]fYX`hc`UhhYbX`HK C`ZJ\W!hc!ZJ\W`gYgg]cbg g\W,YXi `YX`Uhh\Y`VY[]bb]b[`cZh\Y`\\ci fgY''``H\Y`cb`]bY`7 ci fgY`A cXi `Yg`UfY`XYg][ bYX`Zcf`ghi XYbhg`hc a cj Y`h\fci [\`h\Ya `fci [\`m%\&`a cXi `Yg`dYf`XUm''`H\]g`k ]```Ybgi fY`U`VU`Ub\WX`dfYdUfUh]cb`Zcf`h\Y`; `cVU` 7\Ub[Y`\bhYfbg\]d"`Bc`\\can d`Yh]b[`cb`]bY`a cXi `Yg`]b`U`h]a Y`mZJg]cb`\\Ub\`YcdUfX]gY`\\ci fgY`dfc[fYgg]cb \Ub\X`cj YfgYUg`dfYdfUXYbYgg"`

#### 7 ci fgY GWX YXi `Y

#### J]Yk WUgg'ha YHUV'Y

#### H]a YtUV`Y

8 UHY	Hmd Y	7 cbhYbh
K YY_'%'%+': YVfi Ufm!'&% : YVfi Ufm	6`YbXYX	H\]g`Wci gfY`]g`cZYfYX`]b`]bH'bg]jYa cXi g`cjYf`lk ckYY_g''`8i f]b[`lh\]g`ZfghkYY_'h\YfY`]g`cbY`ZUWY!lc! ZUWY`gYgg]cb`cb`A cbXUmž`%+`: YVfi Ufm&\$&\$ fl!*da L''``Ch\Yfk]gY`gh XYbhg`g\ci`X`k cf_'h\fci[\h\Ya cXi`Yg`]b`h\Y]f`ck b`h]a Yžfci[\`m%l&a cXi`Yg\dYf`XUm'``#i]g`]a dcflUbhih\Uhh\Y`gh XYbhg`dUWY`h\Y]f Wca d`Yh]cb`gc`h\YmWUb`Ug_g`ei Ygh]cbg`Zcf`h\Y gYWcbX`ZUWY`fc`ZUWY`gYgg]cb`\Y`X`]b`K YY_&''
K YY_'&.'&( ': YVfi Ufm!'&, : YVfi Ufm		H\]g`Wci gfY`]g`cZYfYX`]b`]bHYbg]j Y`a cXi g`cj Yf`lk c k YY_g"`8i f]b[`lh\]g`gYWcbX`k YY_'lh\YfY`]g`cbY`ZJWY! lc!ZJWY`gYgg]cb`cb`A cbXUmž`&(`: YVfi Ufm&\$&\$ fl!*da Ł"``C h\Yfk ]gY`ghi XYbhg`g\ci `X`Wcbh]bi Y`hc k cf_'lh\fci [\'lh\Y`a cXi `Yg`]b`lh\Y]f`ck b`h]a Yž`fci [\`m %!&`a cXi `Yg`dYf`XUm'``Mci `bYYX`hc`\Uj Y`Wca d`YhYX U```a cXi `Yg`dYf`XUm'``Mci `bYYX`hc`\Uj Y`Wca d`YhYX U```a cXi `Yg`VYZcfY``YUj ]b[`Zcf`mci f`cj YfgYUg k cf_d`UWYa YhYI dYf]YbWY"'  H\Y`: -B 5@: UWY`hc`: UWY`GYgg]cb`k ]```cVWV f`UZhYf mci `fYhi fb`Zfca `cj YfgYUg`UbX`k]```VY`\Y`X`cb`(`A Um &\$&\$`fUZhYfbccbŁ"

## FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

 $\label{eq:decomposition} D`YUgY`Wcbgi`hA~ccX`Yž`U```fYgci~fWfg`UfY`dfcj~]XYX`cb`]bY''`$ 

#### FYWca a YbXYX FYgci fWYg

D`YUgY`Wcbgi `h`A ccX`Yž`U```fYgci fWYg`UfY`dfcj ]XYX`cb`]bY"`

#### 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

Gh XYbhYj U`i Uhj Y`ZYYXVUW\_`]g`[Uh\YfYX`h\fci [\`I BGK fig`A m9I dYf]YbWY"`Gh XYbhg`\Uj Y`U`gc`h\Y cddcfh b]hmhc`dfcj ]XY`ZYXVUW\_`Zcf`YUW\`cb`]bY`Wci fgY`a cXi `Y`UbX`Xi f]b[`h\Y`ZUWY!hc!ZUWY`gYgg]cbg"` Mci f`ZYYXVUW\_`]g`]a dcfhUbhhc`Ybgi fY`h\Uhci f`Wci fgY`XY`]j Yfmi]g`a YYh]b[`gh XYbhbYYXg"`K Y`UfY``cc\_]b[hck UfXg`ghfYUa`]b]b[`h\Y`cb`]bY`Wci fgY`a cXi `Yg`UbX`\Uj Y`i gYX`dUghZYYXVUW\_`hc`]a dfcj Y`XY`]j Yfm'

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

## 5 WUXYa ]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb[ 'hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va ]gg]cbgʻ[ i ]XY`]bYg/ fYj ]Yk ʻcZa Uf\_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/