

# **ARTS2819**

; `cVU`]gUh]cb'UbX'; cjYfbUbW

HYfa "Hk c "#" &\$&%

## 7 ci fgY Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
8YVcfU\^6Uffcg @YU^: Uf]Uq	XYVcfU\V`Z4 i bgk "YXi "Ui	i dcb <sup>·</sup> fYei Ygh	A 6%)	

### GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

## 7ci fgY 8YHJ] g

#### 7 fYX]h'Dc]bhg'\*

#### Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. Dc`lhlWg'UbX' = bhYfbUhlcbU`F Y`Uhlcbg"

#### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

- %" C i h]bY`UbX`Yj U`i UhY`Wca dYh]b[ 'dYfgdYWNj Yg`cb`[ `cVU`]gUh]cb`UbX`[ `cVU``[ cj YfbUbW\ž`XYgWf]V]b[ \_YmXYVUhYg`]b'gh X]Yg`cZWcbhYa dcfUfm[ `cVU``[ cj YfbUbW\"'
- &" 9 I d`U]b`UbX`XYgWf]VY`\ck`[`cVU`]gUh]cb`UZYVWg`X]ZYfYbh'fffY[]a Ygfi]b`[`cVU`dc`]h]Wg''`
- ' " 8 ]gW gg 'h\ Y 'Wca d`YI ]h]Yg 'c Zdc`]Wma U\_]b[ 'Uh'h\ Y '[ `c VU```Yj Y``UbX`YI d`U]b 'h\ Y `fc`Y `c Z\_YmiUWcfg`]b [ `c VU``[ cj YfbUbW'" '
- ("FYWc[b]gY:h\Y:cVgHUWYg:hc:YZZYWf]jY:Wc!cdYfUh]cb:VmgHUh/gž]bhYf[cjYfbaYbHU`:cf[Ub]gUh]cbgždf]jUhY:UWfcfg:UbX:[`cVU`:W]j]:gcW]Ymž:UbX:h\Y:dchYbh]U`:hc:cjYfWcaY:h\YgY:cVgHUWYg":

### HYUW ]b[ 'GhfUhY[ ]Yg'

H\Y`Wci fgY`]g`HJi [\ht\fci [\`U`k YY\_`mk YY\_`mtk c!\ci f``YWfi fY`UbX`cbY!\ci f`ti hcf]U`ž'k ]h\`gca Y`cb`]bY!
ZJW[Wf]i f`ti hcf]U`ž'k ]h\`gca Y`cb`]bY!

## 5ggYgga Ybh

#### 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
7 UgY Gh XmDUdYfg	) \$1	fgYY^A ccX`YŁ	ĬŽ.(
91 Ua ]bUh]cb	) \$1		%ž &

#### 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% 7UgY Gh XmDUdYfg

GHJfhXUhY. Bch5dd`]WUV`Y

8YW]`q.

Ghi XYbhgʻgi Va ]hik cʻ]bX]j ]Xi UʻʻWugYgʻghi XmdUdYfgžʻVUgYXʻcbʻlk cʻgYdUfUhYʻ[cj YfbUbWʻYl Ua dʻYgʻf&ʻl %&\$\$`k cfXgŁ"ʻ9UW WugYʻghi XmdUdYfʻ]gʻk cfh\ '&) i 'cZh\Yʻcj YfU```[fUXYžʻZcfʻUʻhchU``cZ) \$i "'Ghi XYbhgʻa i ghi gyʻh\Y]fʻdUdYfgʻhcʻfYZYWhcbʻYl ]ghjb['fYgYUfW\ž'W ffYbhXYVUhYgʻUbX'dc`]WhX]`Ya a UgʻUWfcggʻh\Y [cj YfbUbWʻYl Ua d`Ygʻ]bʻei Yghjcbʻ]bʻYUW\WUgY"'Ghi XYbhgʻk ]``fYWY]j Yʻk f]hhYbʻZYYXVUW\_žʻUʻWca d`YhYX fi Vf]WUbXʻUʻbi a Yf]WUʻ[fUXYʻh\fci [\`A ccX`Y`k]h\]bʻhk cʻk YY\_gʻcZgi Va ]gg]cb"'

#### 5XX]h]cbU`XYhU]`g.

8YHJ]gfY[UfX]b[gi Va ]gg]cbf]bWi X]b[XUhYgŁUj U]YYcbAccXY

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh]g"gi Va ]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

5ggYgga Ybh& '9l Ua ]bUh]cb

GhufhXuhY. Bch5dd`]WUV`Y

#### 8YHJ]`g.

Ghi XYbhgʻUfYʻfYei ]fYXʻhcʻg]hʻUʻ&!\ci fʻZ]bUʻYl Ua ʻXi f]b[ʻh\Yʻl b]j Yfg]hmfgʻYl Ua ]bUhjcbʻdYf]cX"ʻH\Y
Yl Ua ]bUhjcbʻk ]``Yj Uʻi UhYʻUʻghi XYbhgʻUV]`]hmhcʻh\]b\_ʻgmbh\YhJWU``mUbXʻWf]hJWU``mUvci hh\Yʻa UhYf]U``]bʻh\Y
Wci fgY"ʻ=hfYei ]fYgʻghi XYbhgʻhcʻYb[ U[ Yʻk ]h\ WcbhYbhUWfcggʻh\YʻWci fgY`k\]YʻUg\_]b[ʻh\Ya ʻhcʻXYa cbghfUhY
h\Y]fʻUW\]Yj Ya YbhcZh\Yʻk YY\_`m'YUfb]b[ʻci hWca Yg"ʻ9l Ua ]bUhjcbʻgWf]dhgʻk ]``VYʻa Uf\_YXʻVi hibchfYhi fbYX
hcʻghi XYbhgʻfYWY]j YʻUʻbi a Yf]WU``[ fUXYʻUbXʻWJbʻfYei YghZfca ʻh\YʻGW\cc``cZGcWJU``GWJYbWJgʻhc
gYYʻh\Y]fʻYl Ua ]bUhjcbʻdUdYf"·H\]gʻJgʻh\YʻZ]bU``UggYgga YbhZcfʻh\]gʻWci fgY"

5XX]h]cbU`XYhJ]`g.

 $8YhU]^{\circ}g^{\circ}fY[UfX]b[^{\circ}giVa]gg]cb^{\circ}f]bWiX]b[^{\circ}XUhYgb^{\circ}UjU]^{\circ}UV^{\circ}Y^{\circ}cb^{\circ}AccX^{\circ}Y$ 

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

## 5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj ]Yk '`YWhi fY`fYWcfX]b[g"

# 7ci fgY GWX YXi `Y

### J ]Yk 'WUgg'h]a YhUV'Y

## H]a YtUV`Y

8 UhY	Hmd Y	7 cbhYbh		
K YY_'%'' %'A Um!'('>i bY	@YWi fY	Hcd]W; `cVU`]nUh]cb/; `cVU`; cj YfbUbVW ? YmXYZ]b]h]cbg`UbX`UWcfg		
K YY_^&.^+`>i bY`!^\\\ >i bY	@YWi fY	Hcd]W'l bXYfgHJbX]b[ 'h\Y'; `cVU`'GmgHYa '%		
K YY_" ."%) ">i bY"!"% >i bY	@YWn fY	H\Ya Y. `F YU`]gh/ `@]VYfU``h\Ycf]Yg  Hcd]W`I bXYfghJbX]b[ `h\Y`; `cVU``GmghYa `&` H\Ya Y. `7 f]h]WU``h\Ycf]Yg		
K YY_"(."&%'>i bY"!"&) >i bY	@YWhifY	Hcd]W'H\Y'9j c`i h]cb'cZ'h\Y'; `cVU`'GmghYa '' H\Ya Y.'9a d]fYgž'7 c`cb]nUh]cbž'@cb[ '% h\'W'bh' fm		
K YY_').'&, '>i bY'!'&'>i `m	@YWi fY	Hcd]W'; `cVU`]nUh]cb. &\$h\ Wbh fmZfca UVcj Y H\Ya Y. K K &7 '7 c`X K Uf		
K YY_'*.')	FYUX]b[	FYUX]b[ 'kYY_'! 'bc'WUgg		
K YY_'+.'%&'>i `m'!'%*'>i `m	@YWhifY	Hcd]W; `cVU`]nUh]cb.`&\$h\`WYbh`fmZfca``VY`ck H\YaY.`8YWc`cb]gUh]cb/`]bXYdYbXYbWY		
K_YY_', .'%-'>i `m!'&' '>i `m	@YWi fY	Hcd]W'; `cVU`'; cj YfbUbWfž@Uk ž±bHYfbUh]cbU` CfXYf H\Ya Y. '9bj ]fcba Ybh⁄ '7`]a UHY'7\Ub[Y		
K YY_''&*'>i `m!'' \$'>i `m	@YWnifY	Hcd]W'; `cVU`'9bj ]fcba YbHJ``Dc`]hJVg		
K YY_`%\$.`&`5i [i gh!`* 5i [i gh	@YWhifY	Hcd]W'H\Y'; `cVU`'9Wcbca m H\Ya Y.'≠A : ž'; +ž'; &\$ž'6F =7 G		

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK