

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
HfUWJY 6UfVYf	<u> </u>	D`YUgY`WcbHJWfiVm Ya U]``cf'HYUa g	>%+ž'(\$%5	-',)(\$,%

GWXcc`7cbHJWhi±bZcfa Uhjcb

@cWUhjcb

I BGK 'A YVX.Ub]WU``UbX'A Ubi ZUWi f]b['9b[]bYYf]b[

5]bgk cfh\ 'Vi]`X]b['>%+ž'@Yj Y`'%

5Vcj Y 7 c ZZYY cb 7 Ua di g

<ci fg

-.\$\$-).\$\$da ž'A cbXUm-: f]XUm

ł7`cgYX'cb'di V`]W\c`]XUmgž'GW\cc``gW\YXi `YX'Yj Ybhg'UbX'I b]j Yfg]hmG\i hXck b

K YV

GWXcc`cZA YWXUb]WU`UbX'A Ubi ZUWi f]b['9b[]bYYf]b[

9b[]bYYf]b[Gh XYbhGi ddcfhGYfj]Wg

9b[]bYYf]b[*\DXi glf]U``HfU]b]b[

LBGK 'Gh Xm5VfcUX'UbX'9I WXUb[Y

9a U]`

<u>9b[]bYYf]b['Gh XYbhGi ddcfhGYfj]W</u>yg'-'W ffYbhgh XYbhYbei]f]Yg

• Y"[""Ybfc`a Ybhž'dfc[fYgg]cbž'WUg\"fYei Yghgž'Wci fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg

<u>9b[]bYYf]b['±xi glf]U`'HfU]b]b[</u> '-'±xi glf]U`'lfU]b]b['ei Ygl]cbg

<u>I BGK "Gh Xm5VfcUX"</u>—"gh XmUVfcUX"gh XYbhYbei]f]Yg"fZcf"]bVci bX"gh XYbhgŁ

<u>I_BGK_'9| W.Ub[Y</u>'-'gh XYbhY| W.Ub[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgŁ

I BGK : i h fY'Gh XYbhg'-'dchYbhJU'gh XYbhYbei]f]Yg

• Y"["UXa]gg]cbgžZYYgždfc[fUagžWfYX]hlfUbgZYf

<u>GWXcc``CZZJWY</u>'--'GWXcc``[YbYfU``cZZJWY'UXa]b]ghfUh]cb'Ybei]f]Yg

• B6. h\Y fY`Yj UbhhYUa g``]ghYX UVcj Y a i ghVY WcbHUWYX -

gh XYbhYbei]f]Yg

7ci fgY 8YHJ] g

7 fYX]hDc]bhg (

Gi a a UfmcZh\Y'7ci fgY

H\Y'h\Yg]g'dfcj]XYg'Ub'cddcfhi b]miZcf'mci 'hc'Vf]b['hc[Yh\Yf'Yb[]bYYf]b['df]bW]d'Yg'YuffbYX'cj Yf mci f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd'mh\YgY'df]bW]d'Yg'hc']bbcj Uhjj Y'mgc'j Y'dfcV'Ya g'gi W\'Ug'h\Y XYj Y'cda YbhcZU'gdYW]ZJWXYg][bž'dfcWYgg'UbX#cf'h\Y']bj Ygh][Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc YWg'UfY Wca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhU`ck 'fcca 'Zcf'mci f'WfYUhjj]mž'UbX'h\Y'UWei]g]hjcbž'UbU'mg]g'UbX]bhYfdfYhUhjcb'cZfYgi 'hg''H\YfY'UfY'a i 'hjd'Y'dcgg]V'Y'gc'i hjcbg'cf'WcbWi g]cbg'Uhh\Y'ci hgYhUbX'gi ZZJW]Ybh Wca d'Yl]mhc fYei]fY'U'XY[fYY'cZdfc'YWid'Ubb]b["'H\Y'h\Yg]g'fYei]fYg'mci 'hc'Zcfa i 'UhY'dfcV'Ya g']b gW]Ybh]ZJWcf'Yb[]bYYf]b['hYfa gž'a UbU[Y'Ub'hYW.b]WU'dfc'YWiUbX'Z]bX'gc'i hjcbg'VmUdd'm]b['gW]Ybh]ZJWUbX Yb[]bYYf]b['a Yh\cXg''Mci 'k]``'U'gc'XYj Y'cd'h\Y]f'UV]`]mhc'k cf_']b'U'fYgYUfW\'UbX'XYj Y'cda Ybh Ybj]fcba Ybh''Nci 'a i gh]XYbh]ZmU'gi dYfj]gcf'UbX'dfc'YWidf]cf'hc'Ybfc``]b[']b'h\]g'Wci fgY''H\]g']g'h\Y'Z]fgh Wci fqY'cZh\Y'' 'Wci fqY'h\Yg]g'qlfi Wi fY''

7 ci fgY 5]a g

H\Y'h\Yg]g'dfcj]XYg'Ub'cddcfhi b]hmZcf'h\Y'ghi XYbhhc'Vf]b['hc[Yh\Yf'Yb[]bYYf]b['df]b\Wd'Yg'YUfbYX'cj Yf h\Y]f'dfYj]ci g'mYUfg'cZghi XmUbX'Udd`mh\YgY'df]b\Wd'Yg'hc']bbcj Uhji Y`mgc'j Y'dfcV'Ya g'gi \W.'Ug'h\Y XYj Y`cda YbhcZU'gdY\WZ\WXYg][bž'dfc\W'gg'UbX#cf'h\Y']bj Ygh][Uhjcb'cZU'\mdch\Yg]g''H\Yg]g'dfc'Y\Wg'a i gh VY'\ca d'Yl ž'cdYb!YbXYX'dfcV'Ya g'h\UhiU``ck 'fcca 'Zcf'ghi XYbh\WfVUhjj]hmž'UbX'h\Y'U\\equi]g]hjcbž'UbU'mg]g UbX']b\YfdfYhUhjcb'cZfYgi 'hg''H\YfY'a i ghVY'a i 'h]d'Y'dcgg]V'Y'gc'i hjcbg'cf'\Cb\Wi g]cbg'Uhih\Y'ci hgYhUbX gi \Z\W]Ybh\\ca d'Yl]hm\c'fYei]fY'U'XY[fYY'cZdfc'Y\\fodalbb]b['Zca 'h\Y'ghi XYbh''H\Y'h\Yg]g'fYei]fYg'h\Y ghi XYbhhc 'Zcfa i 'UhY'dfcV'Ya g']b'Yb[]bYYf]b['hYfa gž'a UbU[Y'Ub'Yb[]bYYf]b['dfc'Y\\hiUbX'Z]bX'gc'i hjcbg'Vm Udd`m]b['Yb[]bYYf]b['a Yh\cXg''Ghi XYbhg'U`gc'XYj Y`cd'h\Y]f'UV]`]mhc'k cf_']b'U'fYgYUf\W.'UbX'XYj Y`cda Ybh Ybj]fcba Ybh'

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8 Y j Y`cd 'U 'X Y g][b 'cf 'U 'dfc W gg 'cf '] b j Y g h][U h Y 'U '\ md c h \ Y g] g 'Zc ``ck] b ['] b X i g h f m U b X 'd f c Z Y gg] c b U `
 Y b [] b Y Y f] b ['g H U b { U f j y g'' * Y * h] W U ``a Y h \ c X g " · '
- $\& "7f]h]WU``mfYZYWhicb'`U'gdYW]U`]gh'VcXmicZ_bck``YX[Y'fY``UhYX'hc''h\.Y]f''h\.Yg]g'hcd]W'' \\$
- " 5dd`mgWYbrjZWUbX`Yb[]bYYf]b[a Yh\cXg hc gc`j Y`Ub`Yb[]bYYf]b[dfcV`Ya "
- ("5bU`mgY`XUHU`cV'YWhj Y`mi g]b[ei UbhjHuhj Y`UbXa Uh\Ya UhjWU`a Yh\cXg"
-) "8 Ya cbglfUhY cfu``UbX`k f]hhYb`Wcaaib]WUh]cb`]b`dfcZYgg]cbU``UbX``UmXcaU]bgX`k f]hhYb`Wcaaib]WUh]c'

5ggYgga Ybh

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5ggYgga Ybh'hUg_	K Y][\h	8i Y [.] 8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
H\Yg]g`dfcYWhUbX`fYdcfh	%) ၊	&\$#%#&\$&\$`\$).\$\$`DA	%ž'&ž'' ž'(ž')

5ggYgga Ybh'8YhJ]`g

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Н%

• Dfc YWidfc [fYgg fYdcfhifYdcfh/

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- H\Yg]g'WcbZYfYbWY'dfYgYbHJh]cb/
- :]bU``h\Yg]g`fYdcfh'

5```ghi XYbhUWhji JhjYg`fYei Jf]b[`a Ubi ZJWhi fY`]b`h\Y`A YW\Ub]WJ``UbX`A Ubi ZJWhi f]b[`9b[]bYYf]b[`fA A 9Ł k cf_g\cd`g\ci `X`VY`X]gW ggYX`k]h\ 'h\Y`k cf_g\cd`dYfgcbbY``Uhh\Y`]bWYdh]cb`cZh\Y`k cf_"'H\Y`k cf_g\cd dYfgcbbY``a i gh\Uj Y`h\Y`cddcfhi b]hmhc`UXj]gY`UbX`]bZi YbWY`h\Y`XYg][b`hc`\Y`d`a]b]a]gY`UggYa V`mž a Ubi ZJWhi fY`cf`Z bWh]cbU``dfcV`Ya g"

•

H\Y'k cf_g\cd']g'i gi U``m']b'j Yfm'\][\'XYa UbX"'=\mathref{Trei}]fY'h\Y'k cf_g\cd'hc'a Ubi \mathref{Z}\whi fY'Yei]da Ybh YggYbh]U``hc'mci f'h\Yg]g\tin\Yb'a U_Y'gi fY'h\Uh'mci 'X]g\w gg'mci f'fYei]fYa Ybhg'Ug'YUf'mUg'dcgg]V`Y'k]h\'h\Y K cf_g\cd\@UVcfUhcfmA UbU[Yf''\Mci 'g\ci `X'dfcj]XY'Yb[]bYYf]b['XfUk]b[g'k\]\\\UfY'\alpha[finddfcj YX'Vm h\Y'`\UVcfUhcfmC=\mathref{Im}\Ci 'g\ci 'X'a U_Y'Yj YfmY\alpha[finc a]b]a]gY'h\Y'K cf_g\cd'`cUX'Vma cX]\alpha[b['Yl]gh]b[Yei]da YbhfUh\Yf'h\Ub'Vi]\X]b['\alpha[ca 'bYk \tin\Um'Y\d]b['mci f'XYg][bg'g]a d'Y

UggYgga YbhidYfZcfa UbWYžmci 'UfY'Y`][]V`Y`hc'Udd`m'Zcf'GdYWJU`'7cbg]XYfUhJcb'df]cf'hc'gi Va]hjb['UbUggYgga Ybhicf'g]hjb['Ub'Yl Ua"

I BGK 'bck '\Ug'U': <u>|hhc'G|h#Gi Va |hfi `Y</u>ž'k \]W\ 'a YUbg't\\Uh']Zmci 'UthYa dh'Ub'Yl Ua 'cf'gi Va]h'U'd]YW'cZ UggYgga Ybhž'mci 'UfY'XYWUf]b['mci fgY`ZZ]h'Ybci [\'hc'Xc'gc'UbX'WUbbch`UhYf'Udd`mZcf'GdYW]U` 7 cbg]XYfUh]cb"

: cf`XYhJ]`g`cZUdd`m]b['Zcf`GdYW]U``7cbg]XYfUh]cb`UbX`WcbX]h]cbg`Zcf`h\Y`Uk UfX`cZgi dd`Ya YbhUfm UggYgga Ybhž'd`YUgY`gYY`h\Y`]bZcfa Uh]cb`cb`I BGK 'g`<u>GdYW]U``7cbg]XYfUh]cb`dU[Y</u>"

D`YUgY'bchY'h\Uhghi XYbhg'k]``bch'VY'fYei]fYX'hc'dfcj]XY'Ubm'XcW'a YbhUfm'Yj]XYbWY'hc gi ddcfh'UVgYbWg'Zfca 'Ubm'WUggYg'a]ggYX'**VYWUi gY'cZ'7CJ=8!%** 'di V`]W\YU'h\ 'a YUgi fYg'gi W\'Ug]gc`Uh]cb"'| BGK 'k]``bch'VY']bg]gh]b['cb'a YX]WU`'VW'fh]ZJWUh'Yg'Zfca 'UbmcbY'XYYa YX'hc'VY'U'dcg]h]j Y'WUgYZ cf'k\Yb'h\Ym\Uj Y'fYWcj YfYX"'Gi W\'WYfh]ZJWUh'Yg'UfY'X]ZZJW 'hhc'cVhU]b'UbX'di h'Ub'i bbYWYggUfmghfU]b'cb ghi XYbhg'UbX'a YX]WU`'ghUZZ''

 $5 \ dd^{\wdf} \ duff \ duff$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

 $\label{thm:continuous} IBGK ``Ug`Ub`cb[c]b[`Wcaa] ha Ybhhc`ZcghYf]b[``U`W`hi fY`cZ`YUfb]b[`]bZcfa YX`VmUWUXYa]W]bHY[f]hm'`5``IBGK ``ghi XYbhg`\Uj Y`U`fYgdcbg]V]`]hmhc`UX\YfY`hc`h\]g`df]bW]d`Y`cZUWUXYa]W]bHY[f]hm'UbX`]g`bchhc`YfUhYX`UhIBGK "`B'SY[Sde_SfG@EI[e VWX]`WWSege[`YfZWiadVead[VWSeaXafZWbeS`VbSee[`YfZW_aXSekagdai`ž$

D`U[]Uf]ga `]g`U'hmdY'cZ]bhY``YWi U``h\YZh''±i\Wb'hU_Y'a UbmZcfa gžZfca `XY`]VYfUhY'\W.YUh]b['hc'U\W]XYbhU``m Wcdm]b['Zfca 'U'gci f\W'k]h\ci h'U\w_bck `YX[Ya Ybh''I BGK '\Ug'dfcXi \W'X'U'k YVg]hY'k]h\'U'k YU`h\'cZ fYgci f\W'g'hc'gi ddcfh'ghi XYbhg'hc'i bXYfghUbX'UbX'Uj c]X'd`U[]Uf]ga ž'j]g]h'ghi XYbh'i bgk "YXi "Ui #d`U[]Uf]ga

5 WUXYa]W±bZcfa Uh]cb

7 fYX]hdc]bhg

7 ci fgY'WfYX]hi]g'WU'W'`UhYX']b'I b]hg!C Zi 7 fYX]hifl C 7 L"'H\Y'bcfa U``k cf_`cUX'YI dYWfUh]cb'Zcf'cbY'I C 7]g'UddfcI]a UhY`m'&) '\ci fg'dYf'hYfa "'H\]g']bWi XYg'WUgg'WcbhUWh'\ci fgž'df]j UhY'ghi Xmž'ch\Yf'`YUfb]b[UWfj]]h]Ygž'dfYdUfUh]cb'UbX'h]a Y'gdYbh'cb'U``'UggYggUV`Y'k cf_"

A cgh'Wci fgYk cf_'Wci fgYg'Uhl BGK 'UfY'*'I C 7 'UbX']bj c`j Y'Ub'Ygh]a UhYX'%) \$`\ci fg'hc'Wca d`YhY'''9UW\ Wci fgY']bWi XYg'U'dfYgWf]VYX'bi a VYf'cZ\ci fg'dYf'k YY_'fl\#\ Ł'cZgW\YXi `YX'ZJ\WY!hc!ZJ\W''UbX#cf'cb`]bY WcbhJ\W''5bmUXX]h]cbU`'hja Y'VYmcbX'h\Y'dfYg\fy]VYX'WcbhJ\Wi\ci fg'g\ci `X'VY'gdYbh]b'a U_]b['gi fY'h\Uhmci ii bXYfghJbX'h\Y''YWi fY'a UhYf]U'ž'Wca d'Yh]b['h\Y'gYhUgg][ba Ybhgž'Z fh\Yf'fYUX]b['ž'UbX'fYj]g]b['Zcf UbmYl Ua]bUh]cbg"

Cb!WUa di g WUgg UHYbXUbW

Di V`]WX]gHJbW]b[`WcbX]h]cbg'a i gh'VY`Zc``ck YX`Zcf'U```H' `ZJ\WY!hc!ZJ\WY`WUggYg":Hc`Ybgi fY`h\]gž'cb`m ghi XYbhg`Ybfc``YX`]b`h\cgY`WUggYg'k]```VY`U``ck YX`]b`h\Y`fcca "`7`Ugg`fcghYfg`k]```VY`UhtUW\YX`hc WcffYgdcbX]b[`fcca g`UbX`W]fW``UhYX`Ua cb[``UV`XYa cbghfUhcfg"'Bc'cj Yf!Ybfc`a Ybh]g`U``ck YX`]b`ZJ\WY!hc! ZJ\WY`WUgg"'Ghi XYbhg`Ybfc``YX`]b`cb`]bY`WUggYg`WJb`gk Ud`h\Y]f`Ybfc`a YbhZca 'cb`]bY`hc'ch\Yf`UXX]h]cbU`ž' Vi h``]a]hYXž'bi a VYf`cZcb!WJa di g`WUggYg`VmGi bXUmž'K YY_`%"D`YUgY`fYZYf`hc`mci f`Wci fgYfg`A]\WcgcZh HYUa g`UbX`A ccX`Y`g]hYg`Zcf'a cfY`]bZcfa Uh]cb`UVci h`WUgg`UhhYbXUbWY`Zcf`]b!dYfgcb`UbX`cb`]bY`WUgg gY\W]cbg#J\W]j]h]Yg"

 $\label{thmuh} $$ Mci f'Yu'h' `UbX'h'Y'YU'h' `cZh'cgY']b'mci f'WUgg']g'Wf]hJWU'`m]a dcfhUbh''Mci 'a i ghghUmUh'\ca Y']Zmci 'UfY g]W_'cf'\Uj Y'VYYb'UXj]gYX'hc'gY'Z]gc`UhY'VmBGK_'\YU'h' `cf'[cj Yfba Ybh'Ui h'\cf]hJYg'''7 i ffYbh'U'Yfhg'UbX'U `]gh'cZ\chgdchg'WUb'VY'Zci bX'$

a dcfhUbh'@]b_g