

# **PTRL5019**

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PTRL5019 Term 3, 2020 published at 12-09-2020 //  $\circledcirc$  University of New South Wales, 2020

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- &" 7 f]h]wU``mUbU`mgY`h\Y`i bXYf`m]b[ 'h\Ycf]Ygž'WcbWYdhgž'Uggi a dh]cbg'UbX'Uf[ i a Ybhg'WcbWYfb]b[ 'Zi ]X Zck 'i bXYf`X]ZZYfYbhfYqYfj c]f'WcbX]h]cbg"
- ' "8YjY`cd`fYgYUfW\`UbX`Wcaaib]\Whjcb`g\_]``g`UbX`ghfUhY[]Yg`h\Uhik ci`X`giddcfhih\Y`FYgYUfW\ DfcYWhWcadcbYbhcZh\Y`dfc[fUa"

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K Y YbWci fU[Y'mci'hc'fYhJ]b'U'WcdmcZYj YfmUgg][ba Ybhgi Va]hYX'Zcf'UggYgga YbhZcf'mci f'ck b'fYWcfX Y]hYf']b'YUfXWcdmcf'Y'YWfcb]WZcfa "

5 "UggYgga Ybhg a i gh\Uj Y Ub UggYgga YbhWcj Yf g\YYhUHUWXYX"

#### @UhY Gi Va [gg]cb cZUb 5gg][ ba Ybh

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K Y'i bXYfgHJbX'h\UhiUhih]a Yg'mci 'a Umbch'VY'UV'Y'hc'gi Va ]hiUb'Ugg][ ba Ybh'cb'h]a Yž'UbX'h\Y'GW\cc``k ]``UWWca a cXUhY'UbmZJ]f'UbX'fYUgcbUV'Y'YI hYbg]cb"'K Y'k ci `X'fYWca a YbX'mci 'fYj ]Yk 'h\Y'I BGK 'GdYWJU` 7 cbg]XYfUh]cb'[i]XY`]bYg'—'gYY'gYWfJcb'VY'ck "

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K YʻUg\_ʻh\Uhmci 'd`YUgY WcbhUWh\Yʻ7 ci fgYʻ7 cbj Ybcfʻ]a a YX]UhY`mcbWYʻmci '\Uj Y`Wca d`YhYX'h\YʻgdYW]U` Wcbg]XYfUh]cbʻUdd`]WUh]cbžbc``UhYf'h\UbʻcbY`k YY\_`Zfca 'gi Va ]gg]cb"

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### 5 WUXYa ]W = b Zcfa Uh cb

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±bʻgca Yʻ]bgHJbWYgʻmci fʻZ]bU``Wci fgYʻfYgi `ha UmVY`k]h\\Y`XʻUbXʻbchfY`YUgYXʻcbʻh\YʻI BGK ʻd`UbbYX`XUHY" H\]gʻ]gʻ]bX]WJHYX`VmU`Wci fgYʻ[ fUXYʻfYgi `hcZY]h\Yf.

- @9'-']bX]\\Uh\g`mci'\Uj\Ybch\\cappaca d`Yh\X`cbY`cf'a cfY']h\ag'cZUgg\gga Ybh\cf
- K 8 ]bX]WJhYg h\YfY ]g Ub ]ggi Y k ]h\ cbY cf a cfY Ugg][ ba Ybh/cf
- K 7 K \]WX ]bX]WUhYg`mci `\Uj Y`Udd`]YX`Zcf`GdYWJU``7 cbg]XYfUh]cb`Xi Y`hc`]``bYgg`cf`a ]gUXj Ybhi fY UbX`h\Y`Wci fgY`fYgi `hg`\Uj Y`bchVYYb`Z]bU`]gYX"

7F =7 C G 'Dfcj ]XYf'7cXY. '\$\$\$-,;

# $5W_bck YX[Ya YbhcZ7ci bhfm]$

 $\label{thm:linear} K Y `UW\_bck `YX[Y `th Y `6YXY[U `dYcd `Y `k \c `UfY 'th Y `tfUX]t]cbU `W gtcX] Ubg `cZth Y ``UbXg `cb `k \] WX `I BGK ? Ybg]b[tcb `WUa di g `]g ``cWUhYX"$