

POW these rules: if

use them without understanding and
 later on, understand the first, consider

$$aaaa)(aaa) = aaaaaaaaaa = a^{11} = a^{8+3} .$$

the second: it should be easy to see that we can
 of eight as and a group of three into a group of
 the last formula in () is illustrated by

$$)(a^2)(a^2)(a^2) \\
 a)(aa)(aa)(aa) = aaaaaaaaaa = a^{10} = a^{2 \times 5} ,$$

combined five groups of two as into a single group

important rules are

$$a^0 = 1 ; \quad a^1 = a ; \quad a^{-y} = \frac{1}{a^y}$$

$$\frac{1}{a^y}$$

To understand why the last of these is true, go back to the second
 formula in () and substitute $x = 0$.

(Some of) the above formulae involve the same base to two
 different powers. There are also rules where we have an expression
 involving two bases to the same power:

$$a^x b^x = (ab)^x ; \quad \frac{a^x}{b^x} = \frac{a}{b}^x .$$

Once again you should try to understand why these are true. For
 the first, we know that we can multiply numbers in any order we
 like without affecting the result; so, for example,

$$(ab)^5 = (ab)(ab)(ab)(ab)(ab) = (aaaaa)(bbbbb) = a^5 b^5 .$$

Note that expansions like $(a + b)^x$ are not so easy (not $a^x + b^x$!!)
 and are usually treated by means of the Binomial Theorem.

Although we have so far been thinking of the exponents x, y as
 integers, the same rules apply for any real numbers. A fractional
 power means a root: for example

$$a^{1/2} = \sqrt{a} ; \quad a^{1/3} = \sqrt[3]{a} ; \quad a^{4/5} = (a^4)^{1/5} = \sqrt[5]{a^4} .$$

It is harder to say precisely what is meant by an expression like a^π
 (remember that π is not a fraction) – follow calculus lectures for
 this. Finally, remember that a power expression may be undefined
 for certain values of a . For example, $a^{1/2}$ is meaningless if a is
 negative, and a^{-2} is meaningless if $a = 0$.

Examples.

- $\frac{a^2(ab)^3}{b^4} = \frac{a^2 a^3 b^3}{b^4} = a^5 b^{-1} = \frac{a^5}{b}$.
- $(2c^2 d^5)^3 = 2^3 (c^2)^3 (d^5)^3 = 8c^6 d^{15}$.
- $(x^4 y^5 z^6 y^7 z^8)^{1/4} = (x^4 y^{12} z^{14})^{1/4} = x y^3 z^{7/2} = x y^3 \sqrt{z^7}$.
- $(x^{1 \cdot 2} y^{3 \cdot 4})^2 x^{-5 \cdot 6} = x^{2 \cdot 4} y^{6 \cdot 8} x^{-5 \cdot 6} = x^{-3 \cdot 2} y^{6 \cdot 8}$.

EXERCISES.

Please try to complete the following exercises. Remember that you cannot expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop-in Centre.

1. Following the examples in the first paragraph, write powers in terms of multiplication in order to “explain” the identity $a^9/a^4 = a^{9-4}$.

2. Write the following expressions in terms of products of powers, where each pronumeral occurs once only:

(a) $\frac{x^9(xy^5)^{-2}}{(x^4y)^3}$;

(b) $(a^{2/3}b^{4/5})^6$;

(c) $\frac{(x^5y^3)^{1.3}}{(x^{2.4}y^{3.1})^2}$;

(d) $\frac{(abc^2)^3}{(b^2c)^5} \cdot \frac{(a^3b)^6}{(a^4bc^2)^2}$.

3. Write the following radical expressions in terms of powers, and then simplify them:

(a) _____